Designing Methods

Reinforced Concrete Construction

VOL. 1 JUNE, 1908 No. 2

BUILDINGS

DETAILED DESIGN OF TYPICAL BUILDING

COMPLETE ANALYSIS OF THE STRENGTH OF RECTANGULAR AND "T" SHAPED BEAMS

The design of the typical building is based on the methods outlined in our May bulletin. The analysis for the strength of rectangular and T-shaped beams has been included in this bulletin, owing to the large demand for our former discussion. The matter here given is similar to that which appeared in our 1906 catalogue, but the discussion has been revised and considerably extended.

THE JULY BULLETIN WILL TREAT OF:

Highway bridges and culverts of the flat slab and girder type.

EXPANDED METAL AND CORRUGATED BAR CO.

Suite 925 to 937 Frisco Building

SAINT LOUIS

DESIGN OF A REINFORCED CONCRETE BUILDING.

The following detailed design is given to illustrate methods of computation and to bring out our practice in the application of the formulæ given in the May bulletin.

The design will be based on the following data:

Lot plan; corner lot, 54'-0" x 100'-0".

Column and beam arrangement to be as shown on plan.

Building to be five stories and basement, as indicated on longitudinal section.

Exterior walls, 13" brick walls on spandrel beams.

Allowable soil pressure, 5,000 lbs. per □'.

Roof to be designed for safe live load of 40 lbs. per □'.

Floors to be designed for safe live load of 150 lbs. per □'.

Concrete to be a 1:2:4 mix, good rock or gravel.

All members subjected to flexure will be designed for their breaking or ultimate load, using a factor of 4 on the live load, and 2 on the dead load.

In the design of beams, girders and slabs, allowance will be made for "continuous action." The roof will be figured to carry full live load on all members—the floors to carry full live load on slabs and beams, and 85% of full live load on girders.

In the design of columns the percentage of live load for which they are to be figured is usually prescribed by ordinance. For a building of this kind the average ordinance would permit the columns to be figured to carry 85% of full live load.

Note—The loads specified agree with those used in light warehouse practice, and the design is much heavier than would be required for hotel or office-building construction.

ROOF SYSTEM.

SLABS.

Span: 10'-0" c. to c. of beams.

L. L. 4x40 lbs.=160 lbs.=Ultimate load per □'.

D. L. 2x45 lbs.= 90 lbs.=Ultimate load per □'.

Designing load=250 lbs.=Ultimate load per □'.

The bending moment at the center of span for roof and floor slabs will be taken as $\frac{1}{16} wl^2$ for interior panels and as $\frac{1}{12} wl^2$ for end panels.

All slabs will be figured on a basis of .85% of reinforcement.

INTERIOR PANELS.

Ultimate moment=$\frac{1}{16}\times 250\times 10^2$= 1,560 ft. lbs.
=18,720 in. lbs.

For this percentage of reinforcement $M_o=370\ bd^2$. (See formula 1, page 8.)

$b=12''$, therefore $M_o=4{,}440\ d^2=18{,}720''$ lbs., from which d=2.06''. $q=.0085\ bd=.0085\times12\times2.06=0.21\square''$.

For practical reasons, it is not advisable to make the slab less than $3\frac{1}{2}''$ thick, in which case d may be taken equal to $2\frac{1}{2}''$. We will use this thickness of slab and reduce the amount of reinforcement as found above in the ratio $\frac{2.06}{2.5}$; using 0.173 sq. inches of metal.

Make slab $3\frac{1}{2}''$ thick, $\frac{1}{3}''$ corr. sq. N. S. 8'' cts.

Since the above design assumes reverse bending moment over the beams, the same amount of reinforcement should be placed in the top of slab at that point. This reinforcement will consist of $\frac{1}{3}''$ corr. sq. N. S. 8'' cts. extending to the quarter points of the panels.

It is desirable to have transverse reinforcement in slabs to prevent temperature or shrinkage cracks, and also to act as distributing bars.

END PANELS.

$M_o=\frac{1}{12}\ wl^2=24{,}800$ inch lbs. (ult.)

The ultimate resisting moment of a $3\frac{1}{2}''$ slab, assuming $d=2\frac{1}{2}''$, reinforced with .85% reinforcement$=M_o=370\ bd^2=370\times12\times2.5^2=27{,}750$ in. lbs. It is desirable to keep the thickness of slab uniform, and to illustrate a different method we will determine the amount of steel by the formula $M_o=Fq\times.86d=24{,}800''$ lbs.$=50{,}000\times q\times.86\times2\frac{1}{2}=24{,}800$, from which $q=0.23\square''=\frac{1}{3}''$ corr. squares, N. S. 6'' cts.

BEAMS.

We will design a typical interior beam; B5.

Span of beams, c to c of supports,=18'-0''.

Distance between beams, c to c,=10'-0''

Ultimate load on beam:

Live Load	$=40\times10\times18=7{,}200$ lbs.;	$\times4=28{,}800$ lbs.
D. L. Slab	$=45\times10\times18=8{,}100$ lbs.;	$\times2=16{,}200$ lbs.
D. L. Beam	$=10\times10\times18=1{,}800$ lbs.;	$\times2=\ \ 3{,}600$ lbs.
	Actual, 17,100 lbs.;	Ult., 48,600 lbs.

$M_o=\frac{1}{12}\ Wl=\frac{1}{12}\times48{,}600\times18=72{,}900'$ lbs.$=874{,}800''$ lbs. (ult.)

In beams and girders we will use 1.3% reinforcement, and design by formula 2, page 8; $M_o=570\ bd^2$. Assuming $b=9''$, we have $M_o=570\times9d^2=874{,}800$, from which $d=13.1''$.

$q=.013\ bd=.013\times9\times13.1=1.54$ sq. inches.

d=distance from top of beam to plane of metal.

Make beam 9''x15$\frac{1}{2}$''; 4-$\frac{5}{8}''$ corr. squares N. S.; bend up two bars and extend to quarter points of beam in adjacent panel. This design gives about 2'' of fireproofing on the under side of the bars.

SHEARING PROVISIONS.

For complete discussion see page 12.

It will be assumed that the concrete will carry safely 50 pounds of vertical shear per square inch of cross-section of beam.

We then have $V_c = bd\ v_c = 9 \times 13.1 \times 50 = 5{,}900$ lbs. Stirrups in roof beams will be ¼" corrugated squares, bent in U-shape; allowed working stress=16,000 lbs. per sq. in. Then $P = 2 \times 0.06 \times 16{,}000$ lbs.= 1,820 lbs.=total allowable stress in one stirrup.

To determine stirrup spacing, use formula on page 14.

$$y = \frac{0.86\ d\ P}{V - V_c} = \frac{0.86 \times 13.1 \times 1820}{V - 5900} = \frac{20500}{V - 5900}$$

The spacing at any point may be determined by substituting the value of the external shear, V, at that section in the above.

The following table illustrates the method:

Distance from End of Beam.	Size of Stirrups.	Vert. Ext. Shear $=V$.	V_c	$V - V_c$	Spacing$=y = \frac{0.86\ d\ P}{V - V_c}$
0'	¼"	8550 lbs.	5900 lbs.	2650 lbs.	8"
1'	Corrugated	7600 lbs.	5900 lbs.	1700 lbs.	12"
2'	Squares	6650 lbs.	5900 lbs.	750 lbs.	27"
4'	New Style.	4750 lbs.	5900 lbs.	....	...

Beams in end panels to be figured in a similar manner, using formula $M_o = \frac{1}{10}\ Wl$.

GIRDERS.

Typical interior girder, G_4.

The actual dead and live load concentrated at the center of the girder is 17,100 pounds, corresponding to an ultimate load of 48,600 pounds, not including the weight of the girder itself.

Ult. moment at center $M = \frac{1}{6}\ Wl$. (For a beam simply supported this would be $\frac{1}{4}\ Wl$. for center load.)

$M_o = \frac{1}{6} \times 48{,}600 \times 20 =$ .... 162,000 ft. lbs. (ult.)

Assume girder weighs 250 lbs. per ft., then

M due to its weight$= \frac{1}{12} \times 250 \times 20^2$, times 2= 16,700 ft. lbs. (ult.)

TOTAL ULTIMATE MOMENT,= 178,700 ft. lbs. (ult.)

=2,144,400 in. lbs. (ult.)

Assume $b = 12''$. As before $570 \times 12 \times d^2 = 2{,}144{,}400$ from which $d = 17.7''$. $q = .013\ bd. = 2.76$ sq. in.

Make girder 12"x20½"; 5-¾" corrugated squares N. S.

The same amount of reinforcement should be used in the top of the girder over the supports. This will be provided for by bending up two bars in each girder, and using an additional ¾-inch bar, placed as shown on drawing.

SHEARING PROVISIONS.

Neglecting the weight of the girder, the shear is constant from the column to center of span.

$$V=\frac{17,100}{2}=8,550 \text{ lbs., neglecting weight of girder.}$$

$$V_c=12\times 17.7\times 50=10,620 \text{ lbs.}$$

Although at the allowed stress of 50 lbs. per sq. in. in the concrete, no stirrups would be required, it is essential that at the columns and where the beams frame into girders, that stirrups be provided to distribute the concentrated loadings at these points.

GIRDERS IN END PANELS.

Design girders $G3$ in a similar manner, using the formula $M=\frac{1}{6}\ Wl$. (The formula $\frac{1}{8}\ Wl$ and $\frac{1}{6}\ Wl$ are for one concentrated load at center of span.)

In the design of end beams and girders it is desirable to increase the percentage of reinforcement, keeping same dimensions as for interior beams, rather than make the percentage of reinforcement constant and change the size of the beams.

FLOOR SYSTEM.

SLABS, INTERIOR PANELS.

Span=10′-0″ c to c beams.

L. L. 150 lbs.×4=600 lbs.=ult. load per □′.
D. L. 60 lbs.×2=120 lbs.=ult. load per □′.

720 lbs.=ult. load per □′.

$$M_o=\frac{1}{16}\ wl^2=\frac{1}{16}\times 720\times 10^2= 4,500 \text{ ft. lbs. (ult.)}$$
$$=54,000 \text{ in. lbs. (ult.)}$$

As before $M_o=370\ bd^2=54,000$, from which $d=3.5$ in.

$q=.0085\times 12\times 3.5=0.360$ sq. inches per foot width.

Make slab 4½″ thick; ½″ corrugated rounds 6½″ cts.

Design end panels by formula $M_o=\frac{1}{12}\ wl^2$.

BEAMS.

Typical interior beam, $B5$.

Span, c to c of supports,=18′-0″.

Spacing of beams, c to c,=10′-0″

Actual Load on Beam.		× Factor =	Ultimate Load on Beam.
L. L.,	150×10×18=27,000 lbs.	× 4 =	108,000 lbs.
D. L. (Slab),	60×10×18=10,800 lbs.	× 2 =	21,600 lbs.
D. L. (Beam),	15×10×18= 2,700 lbs.	× 2 =	5,400 lbs.
Totals..................	40,500 lbs.		135,000 lbs.

$$M_o = \tfrac{1}{12}\ Wl = \tfrac{1}{12} \times 135{,}000 \times 18 = \ 202{,}500 \text{ ft. lbs.}$$
$$= 2{,}430{,}000 \text{ in. lbs. (ult.)}$$

Using 1.3 per cent of reinforcement, $M_o = 570\ bd^2 = 2{,}430{,}000$.
Assuming $b = 12''$; $d = 18.9''$ and $q = 2.96\square''$.

Make beam 12''x21''; 4-⅞'' corrugated squares, N. S.

Bend up two bars in each beam.

Figure beams in end panels similarly, using formula $M = \frac{1}{10}\ Wl$.

SHEARING PROVISIONS.

$$V \text{ at end} = \frac{40{,}500}{2} = 20{,}250 \text{ lbs.}$$

$$V_c = 12 \times 18.9 \times 50 = 11{,}350 \text{ lbs.}$$

Using "U"-shaped stirrups, ⅜'' corrugated rounds, we have:

$P = 2 \times 0.11 \times 16{,}000 \text{ lbs.} = 3{,}520 \text{ lbs.}$

$y \text{ at end} = \dfrac{.86\ d\ P}{V - V_c} = \dfrac{.86 \times 18.9 \times 3{,}520}{20{,}250 - 11{,}350} = 6\tfrac{1}{2}''.$

y, three feet from end$=26''$.

GIRDERS.

Typical interior girder, $G4$.

All girders in floor system will be figured to carry 85% of the total live load.

ACTUAL CONCENTRATED LOAD ON GIRDER.	× FACTOR =	ULTIMATE LOAD.
L. L. 0.85×27,000 .... =22,900	× 4 =	91,600 lbs.
D. L. (Slab) 10,800	× 2 =	21,600 lbs.
D. L. (Beam) 2,700	× 2 =	5,400 lbs.
TOTALS 36,400		118,600 lbs.

M_o concentrated load$= \frac{1}{6}\ Wl = \frac{1}{6} \times 118{,}600 \times 20 = \ldots\ldots 395{,}000$ ft. lbs.

The girder will weigh approximately 300 lbs. per ft.

$M_o = 2 \times M$ due to weight of girder$= 2 \times (\frac{1}{12} \times 300 \times 20^2) = 20{,}000$ ft. lbs.

TOTAL .... 415,000 ft. lbs.

$$M_o = 570\ bd^2 = 4{,}980{,}000 \text{ in. lbs. (ult.)}$$

Assume $b = 14''$; $d = 25''$ and $q = 4.55\ \square''$.

Make girder 14''x28''; 6-⅞'' corrugated squares, N. S.

Bend up three bars in each girder.

SHEARING PROVISIONS.

V may be assumed constant and equal to $\frac{36,400}{2}=18,200$ lbs.

$V_c=14\times25\times50=17,500$ lbs.

$V-V_c=700$ lbs., indicating that the concrete alone is capable of carrying the shear at the allowed stress. As before stated, it is desirable to have stirrups to distribute concentrated loadings, and they will be provided as indicated on drawings.

COLUMN DESIGN.

TYPICAL INTERIOR COLUMN.

Contributory area=18'×20'=360 sq. ft.

The unit stress in the concrete will be taken as 600 lbs. Referring to diagram on page 18, the average permissible stress per sq. inch, using 1% reinforcement, is found to be 690 lbs.

This figure will be used to determine the required area—one inch of concrete will be added all around for fire-proofing.

As stated, the live load on columns will be taken as 85% of full live load.

Story.	Story Loads.	Column Load=P.	COLUMN SECTION.			
			Area Req'd $=P\div690$.	External Dimensions.	Vertical Reinforced.	Hooping.
5th Height=11'-0"	L. L.....14400 D. L.....28800 Col. 1100 44300	44300	64□"	12"x12"	4—½" C. R.	¼" C. R. 12" cts.
4th Height=11'-0"	L. L.....45900 D. L.....39600 Col. 2900 88400	132700	193□"	16"x16"	4—¾" C. R.	¼" C. R. 12" cts.
3d Height=11'-0"	L. L.....45900 D. L.....39600 Col. 4500 90000	222700	323□"	20"x20"	8—¾" C. R.	¼" C. R. 12 cts.
2d Height=11'-0"	L. L.....45900 D. L.....39600 Col. 6000 91500	314200	455□"	24"x24"	8—⅞" C. R.	¼" C. R. 12" cts.
1st Height=15'-0"	L. L.....45900 D. L.....39600 Col. 10500 96000	410200	693□"	27"x27"	8–1" C. R.	¼" C. R. 12" cts.
Basement Height=10'-0"	L. L.....45900 D. L.....39600 Col. 9500 95000	505200	732□"	29"x29"	8—1⅛" C. R.	¼" C. R. 12" cts.

TYPICAL WALL COLUMN.

Story.	Story Loads.	Column Load=P.	COLUMN SECTION.			
			Area Req'd $=P\div 690$.	External Dimensions.	Vertical Reinforced.	Hooping.
5th Height=11'-0"	L. L.... 7200 D. L.....14400 Col. 3500 25100	25100	36□"	*13"x24"	4—¾" C. R.	¼" C. R. 12" cts.
4th Height=11'-0"	L. L....22950 D. L....19800 Wall ...19000 Col. 3500 65250	90300	131□"	13"x24"	4—¾" C. R.	¼" C. R. 12" cts.
3d Height=11'-0"	L. L....22950 D. L....19800 Wall ...19000 Col. 3500 65250	155600	225□"	13"x24"	6—¾" C. R.	¼" C. R. 12" cts.
2d Height=11'-0"	L. L....22950 D. L....19800 Wall ...19000 Col. 4100 65850	221400	320□"	15"x24"	6—⅞" C. R.	¼" C. R. 12" cts.
1st Height=15'-0"	L. L....22950 D. L....19800 Wall ...19000 Col. 7000 68750	290200	422□"	19"x24"	6—1" C. R.	¼" C. R. 12" cts.
Basement Height=10'-0"	L. L....22950 D. L....19800 Wall ...27000 Col. 6000 75750	366000	530□"	24"x24"	6—1⅛" C. R.	¼" C. R. 12" cts.

*Wall columns are made of uniform width throughout to simplify brick work.

FOOTINGS.

TYPICAL INTERIOR FOOTING.

Load at base of column ......=505,200 lbs.
Approximate weight of footing= 25,000 lbs.

530,200 lbs.

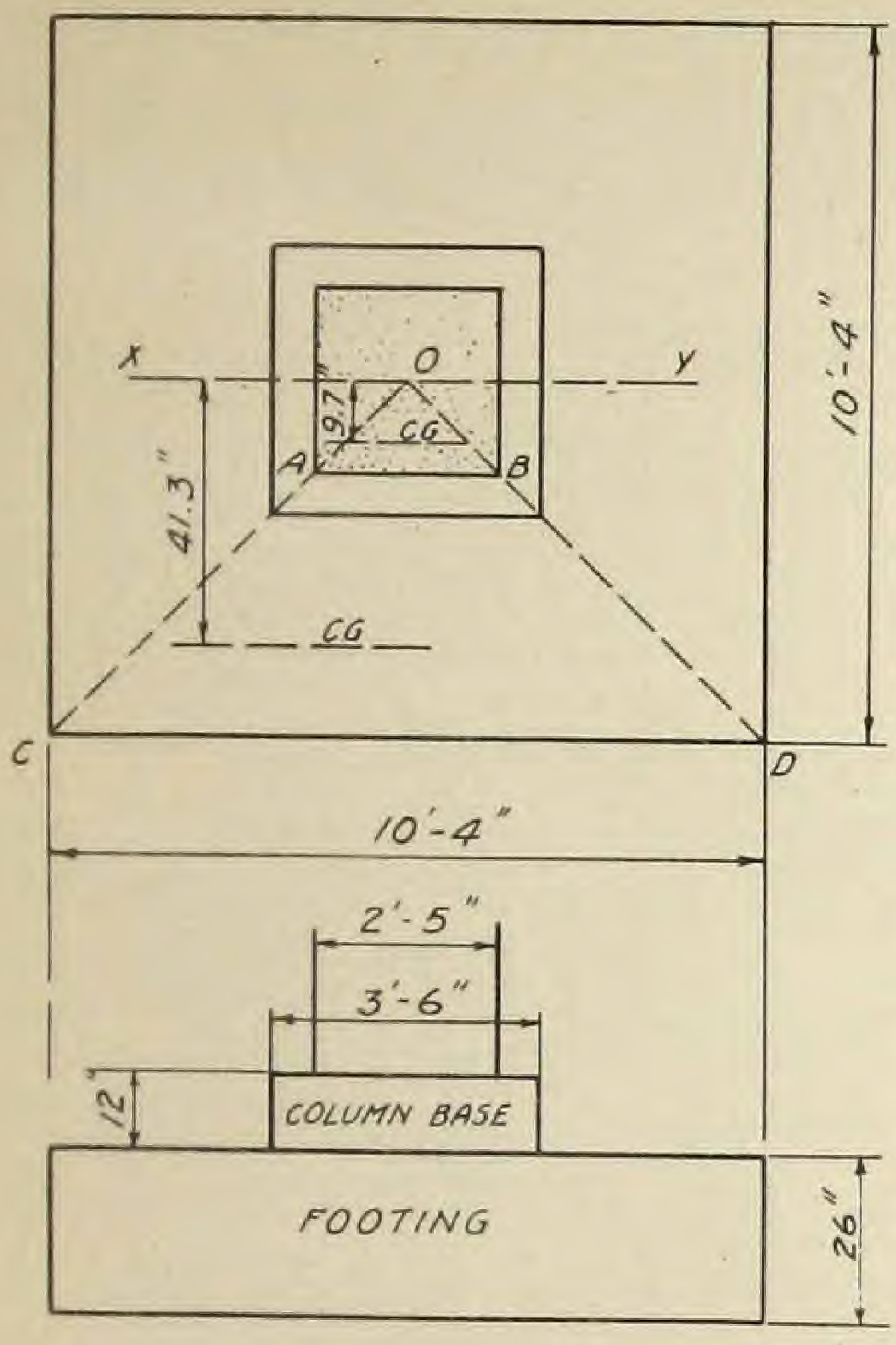
Required area$=\frac{530,200}{5,000}=106$ sq. ft.

Make footing 10'-4"x10'-4".

In the design of the footing we will not allow the vertical shearing stress to exceed 125 lbs. per sq. in.

Required depth of footing then$=\frac{505,200}{4\times 29\times 125}=35''$.

As the shear decreases toward the edge of the footing, and this depth of footing is not required to resist the bending moment developed, we will step off the footing, as shown.



The upward pressure on the triangle C-O-D and the downward pressure on the triangle A-O-B are each equal to one-fourth of the column load, neglecting the weight of the footing.

The maximum bending moment is in the plane x-y, and may be obtained by taking moments about this line. Let a=distance from x-y to center of gravity of triangle C-O-D and b=distance to center of gravity of triangle A-O-B. Then $M=\frac{505{,}200}{4}(a\text{-}b)=\frac{505{,}200}{4}(41.3''-9.7'')=3{,}990{,}000$ in. lbs.

Using a factor of safety of 3, the ultimate moment equals M_o= 11,970,000 inch lbs. This moment is resisted by a section in the plane x-y, the width considered available, may be taken equal to the side of the column plus 1½ times the depth of the footing=29″+(1½×23″)=5′-3½″.

The column base is used to distribute the load over the footing and its dimensions are determined from shearing considerations only. The footing and column base must be built monolithic.

The moment per foot width of section=11,970,000÷5.29=2,260,000 inch lbs. (ult.)

For footings it is necessary to have a comparatively large amount of concrete to resist shear. We will therefore use .85% reinforcement.

$M_o=370\ bd^2=2{,}260{,}000$ for $b=12''$, whence $d=22.5''$.

$q=.0085\times12\times22\frac{1}{2}=2.29$ sq. inches per foot width.

Make $d=23''$ and use 1″ corr. sq. new style, 5″ cts. both ways in footing.

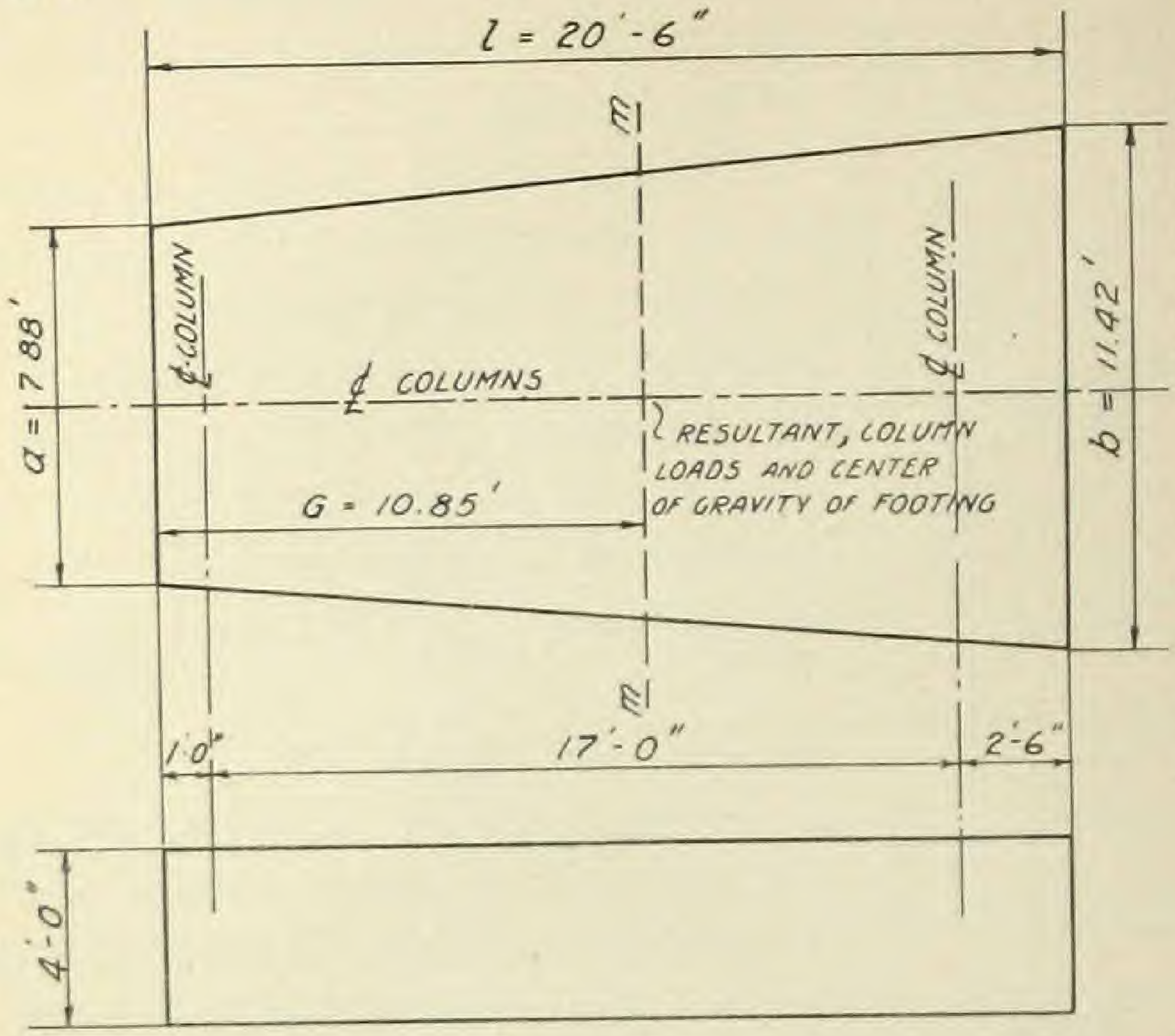
COMBINED FOOTINGS.

Since the footings are not to extend beyond the inside lot line, it will be necessary to use combined footings on this side of the building. The combined footing acts as a beam resisting the upward reaction of the soil, and supported at the ends by the columns. By making the center of gravity of the footing plan correspond with the resultant of the column loads, a uniform soil-pressure is produced.

Load on wall column	366,000 lbs.
Load on interior column	505,200 lbs.
TOTAL	871,200 lbs.
Assuming footing to be 4′-0″ thick, its weight would be about	120,000 lbs.
TOTAL ON SOIL	991,200 lbs.

$$\text{Area required}=\frac{991{,}200}{5{,}000}=198 \text{ sq. feet}$$

Since the column loads are unequal, the footing will have a trapezoidal shape, and by fixing its length, since we have the area, we can determine the length of the two parallel sides for conditions of uniform load.



Let G = distance from small end to center of gravity of footing and to the resultant of column loads.

If A = area of trapezoid, we can determine the ends "a" and "b" from the following formula:

$$b=\frac{2A}{l}\left(\frac{3G}{l}-1\right)$$

The average width $=\frac{a+b}{2}$ must equal $\frac{A}{l}=\frac{198}{20.5}=9.65$ feet, or $a=19.3-b$.

Substituting in the formula we find $b=11.42$ feet, $a=7.88$ feet.

The load on soil equals 5,000 lbs. per sq. foot; the weight of footing is 600 lbs. per sq. foot; upward reaction equals 4,400 lbs. per sq. foot.

To design footing, we will find the moment on a strip 12" wide. Using a factor of 3 on the bending moment, we have

$$M_o=3\left(\tfrac{1}{8}wl^2\right)=3\left(\tfrac{4400}{8}\times17^2\times12\right)=5{,}720{,}000 \text{ in. lbs. (ult.)}$$

By means of table No. 1, page 23, average rock concrete, we find that the section corresponding to this moment requires $d=36''$ and $q=3.70\square''$.

Since we have fixed upon the total depth of the footing as 48", we will make $d=44''$ and use $\frac{36}{44}\times3.70''=3.03\square''$ of metal per foot width.

Make footing 48" deep, d=44", and use 1" corr. squares, N. S., 4" cts. in top of footing.

The distance, c to c given, is on the line m-m through the center of gravity of the footing.

At the ends of the footing the portions under the columns act as transverse beams. We will figure the reinforcement required for this cantilever action at the larger end, under interior column. Ultimate moment $=M_o=\frac{505200}{2}\times\frac{11.42}{4}\times12\times3=26{,}000{,}000$ in. lbs. This will be assumed to be distributed over a width of five feet.

M_0 per foot width equals 5,200,000 in. lbs. (ult.); this is practically the same as the reverse moment in the longitudinal direction and the same reinforcement will be used.

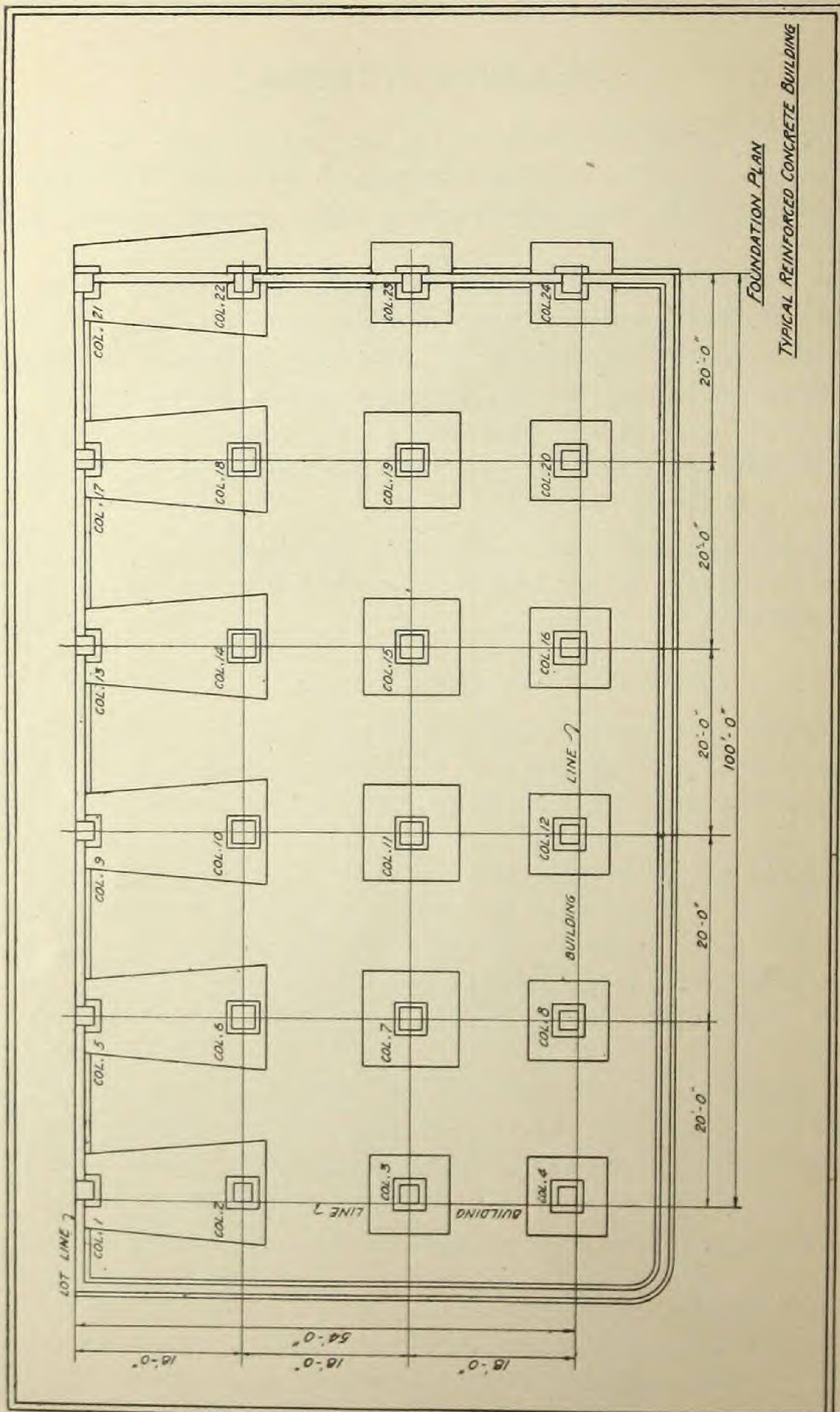
SHEARING STRESSES.

Limiting the vertical shear to 125 lbs. per sq. inch, the minimum depth of footing under interior column will be found to be 35 in.; as the footing is 48 in. deep the punching shear at the end need not be considered.

The wall column will require investigation, since only three sides may be considered in determining the intensity of the shear. To bring this within safe limits, and also to avoid high stresses at the edge of the footing, the column will be arranged as shown on drawing.

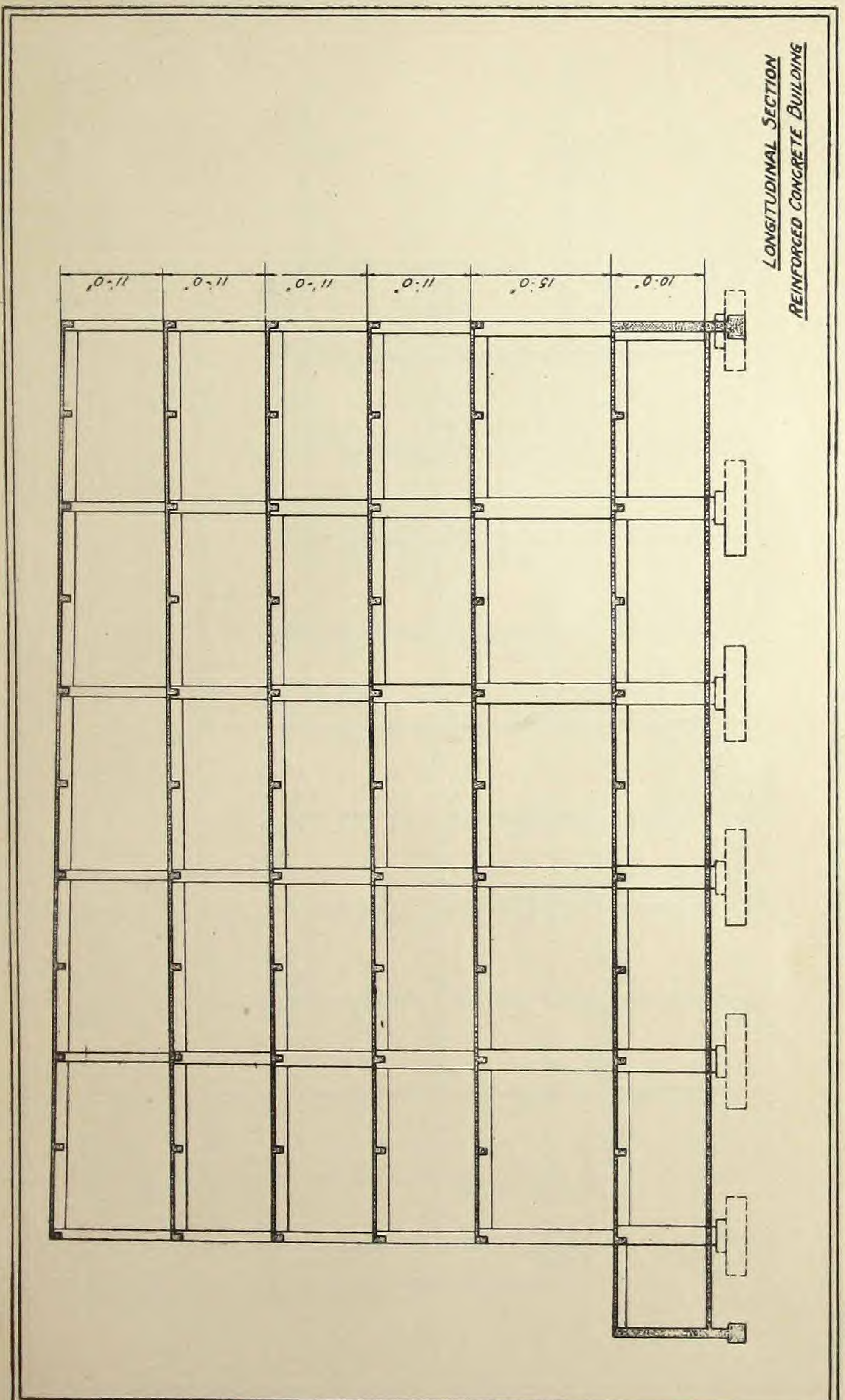
In order to avoid any possibility of the concrete shearing along diagonal planes at the ends, horizontal and vertical tie bars should be introduced, as shown on detail.

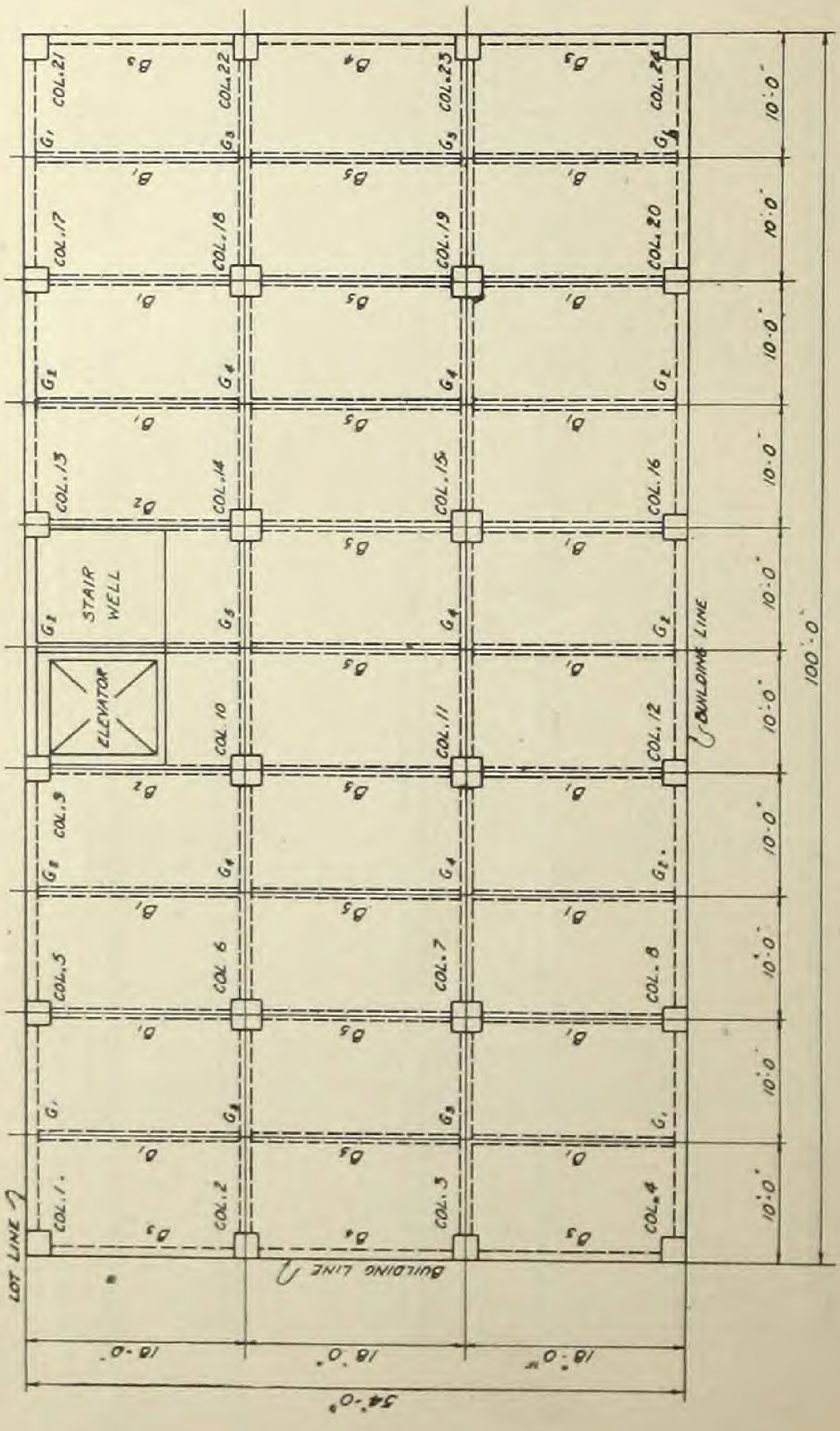
Note—No provisions have been made in this building for resisting wind stresses, as, owing to its low height in comparison to its width, such stresses may be neglected.



FOUNDATION PLAN

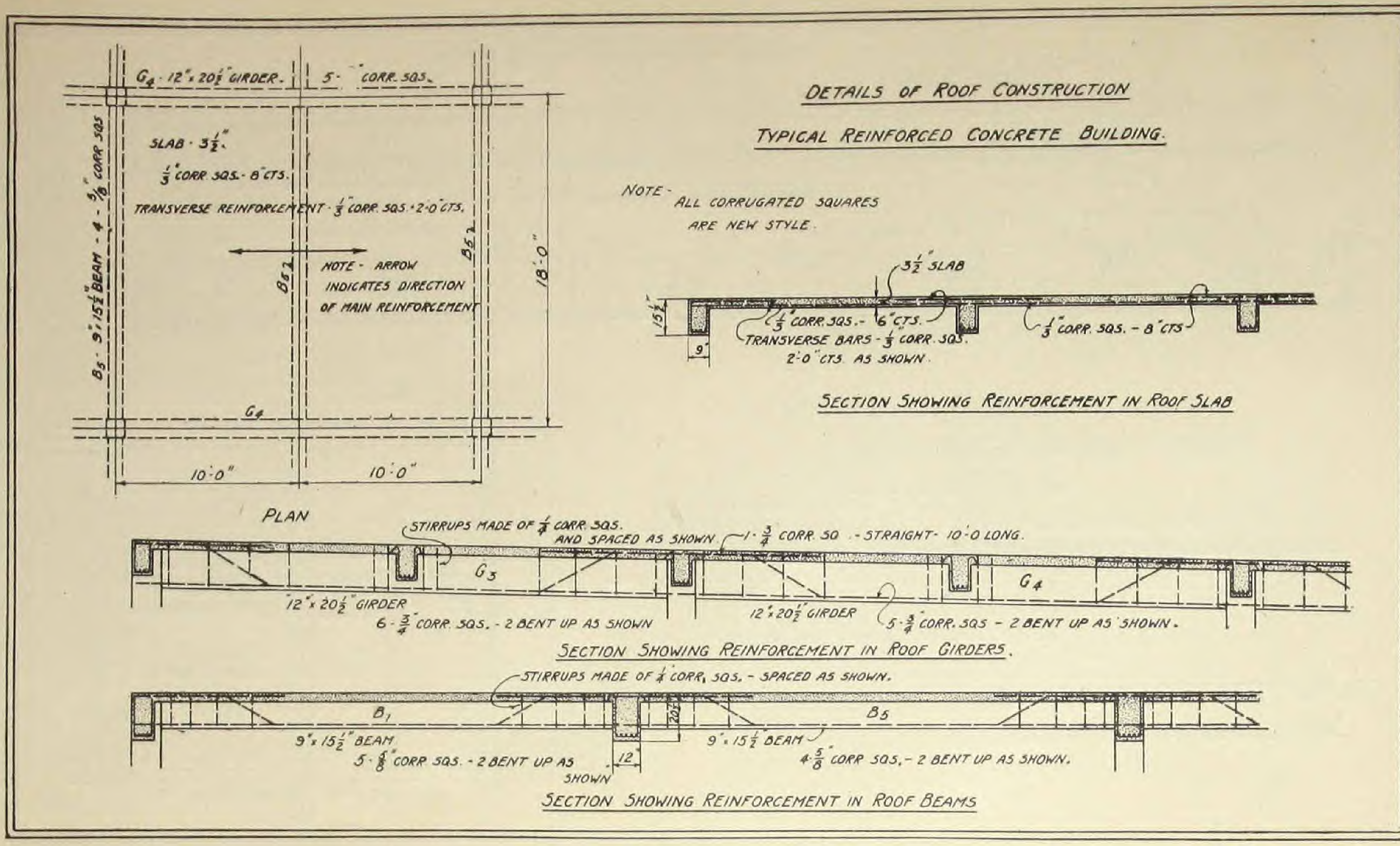
TYPICAL REINFORCED CONCRETE BUILDING

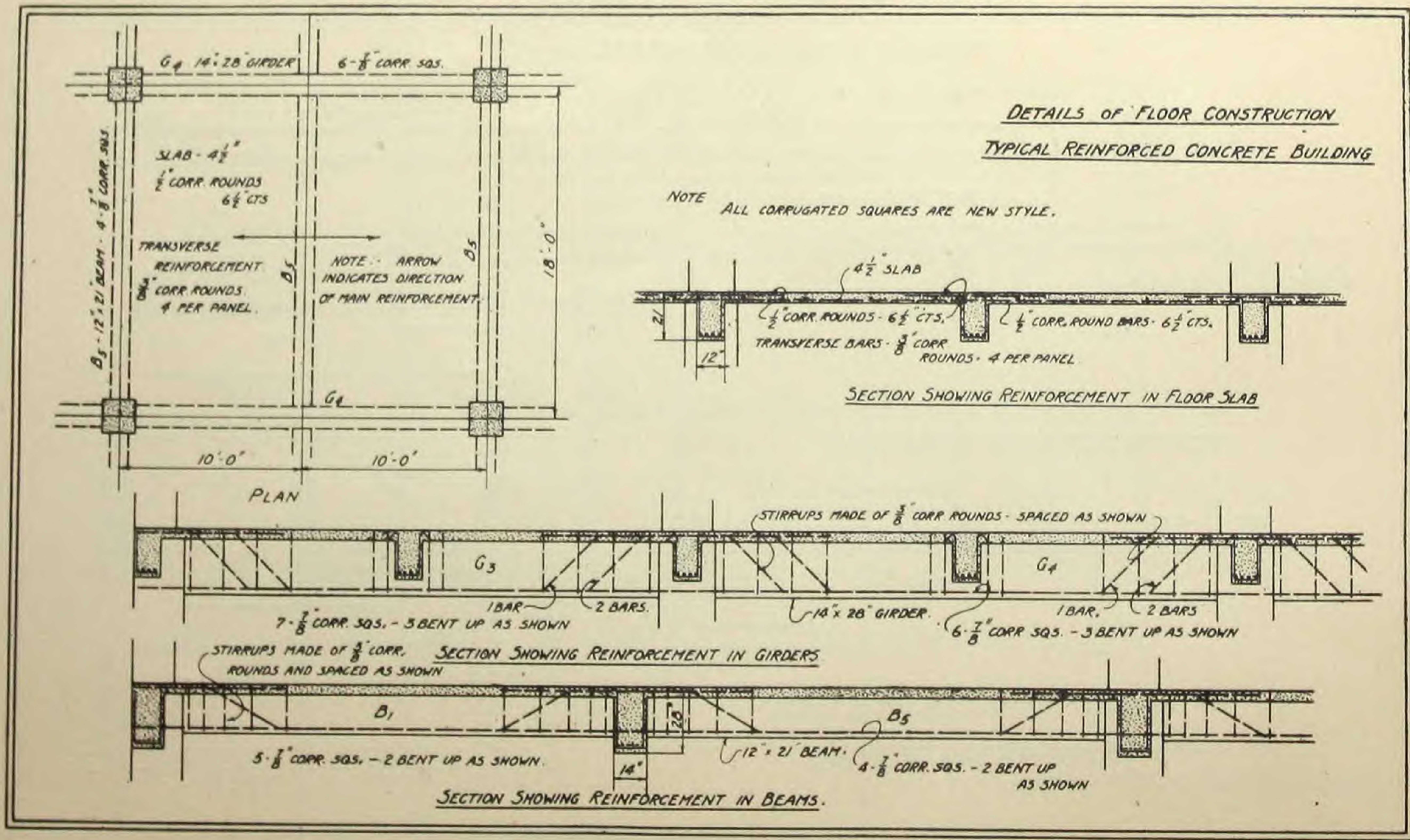

LONGITUDINAL SECTION
REINFORCED CONCRETE BUILDING
11'-0"
11'-0"
11'-0"
11'-0"
15'-0"
10'-0"

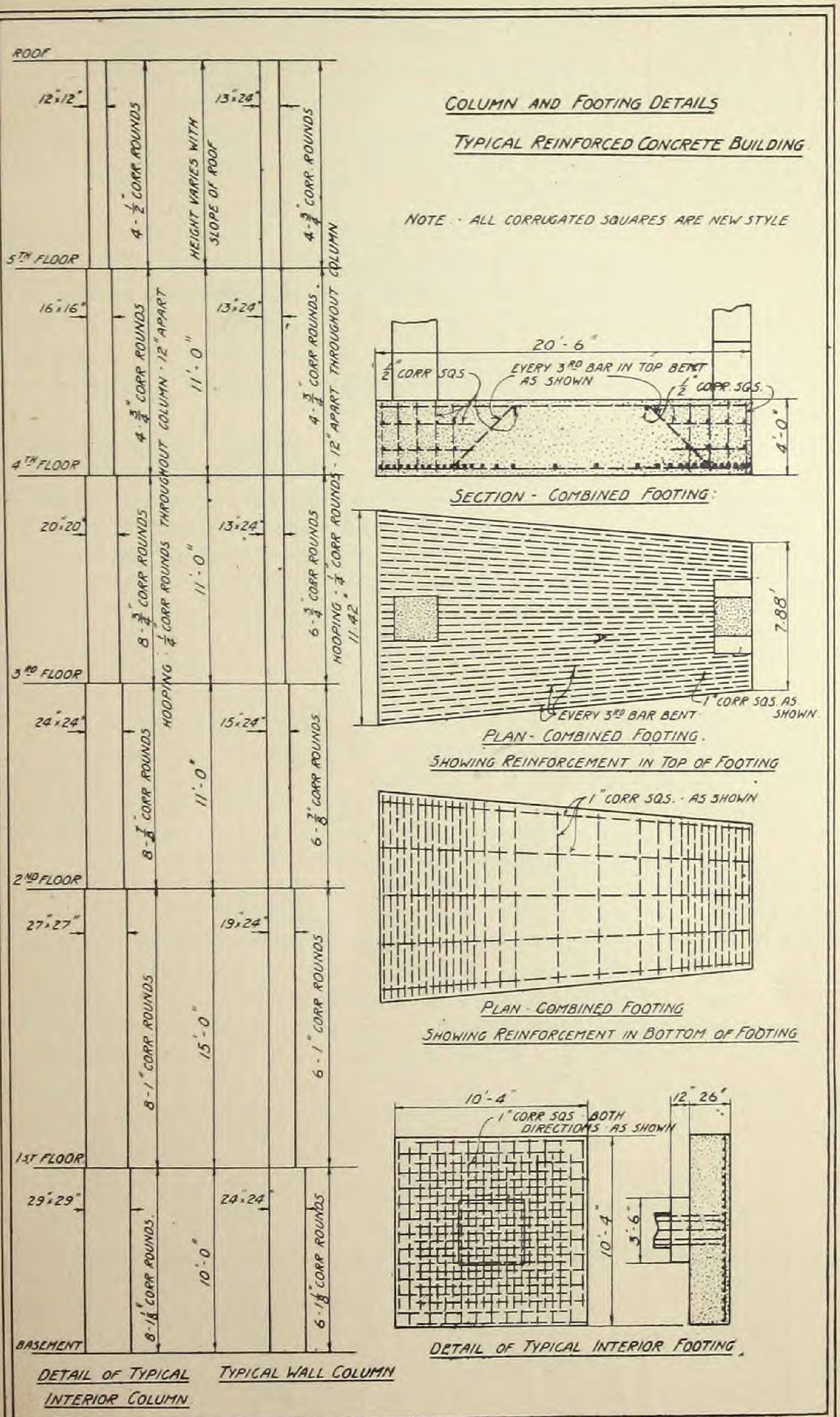


TYPICAL FRAMING PLAN.

TYPICAL REINFORCED CONCRETE BUILDING.


DETAILS OF ROOF CONSTRUCTION
TYPICAL REINFORCED CONCRETE BUILDING.
NOTE - ALL CORRUGATED SQUARES ARE NEW STYLE.
G4 · 12" x 20½" GIRDER. 5 · CORR. SQS.
B5 · 9" x 15½" BEAM - 4 - 5/8" CORR. SQS.
SLAB · 3½".
⅓" CORR. SQS. - 8" CTS.
TRANSVERSE REINFORCEMENT · ⅓" CORR. SQS. · 2'-0" CTS.
NOTE - ARROW INDICATES DIRECTION OF MAIN REINFORCEMENT
B52
G4
10'-0"
10'-0"
18'-0"
PLAN
3½" SLAB
15½"
9"
⅓" CORR. SQS. - 6" CTS.
TRANSVERSE BARS - ⅓" CORR. SQS. 2'-0" CTS. AS SHOWN.
⅓" CORR. SQS. - 8" CTS.
SECTION SHOWING REINFORCEMENT IN ROOF SLAB
STIRRUPS MADE OF ¼" CORR. SQS. AND SPACED AS SHOWN.
1 · ¾" CORR. SQ. - STRAIGHT - 10'-0 LONG.
G3
G4
12" x 20½" GIRDER
6 · ¾" CORR. SQS. - 2 BENT UP AS SHOWN
12" x 20½" GIRDER
5 · ¾" CORR. SQS - 2 BENT UP AS SHOWN.
SECTION SHOWING REINFORCEMENT IN ROOF GIRDERS.
STIRRUPS MADE OF ¼" CORR. SQS. - SPACED AS SHOWN.
B1
B5
9" x 15½" BEAM
5 · ⅝" CORR. SQS. - 2 BENT UP AS SHOWN
12"
20½"
9" x 15½" BEAM
4 · ⅝" CORR. SQS. - 2 BENT UP AS SHOWN.
SECTION SHOWING REINFORCEMENT IN ROOF BEAMS


DETAILS OF FLOOR CONSTRUCTION
TYPICAL REINFORCED CONCRETE BUILDING
NOTE ALL CORRUGATED SQUARES ARE NEW STYLE.
G4 14"x28" GIRDER
6-7/8" CORR. SQS.
SLAB - 4 1/2"
1/2" CORR. ROUNDS 6 1/2" CTS
TRANSVERSE REINFORCEMENT 3/8" CORR. ROUNDS. 4 PER PANEL.
NOTE:- ARROW INDICATES DIRECTION OF MAIN REINFORCEMENT.
B5 - 12"x21" BEAM - 4-7/8" CORR. SQS.
B5
G4
18'-0"
10'-0"
10'-0"
PLAN
4 1/2" SLAB
1/2" CORR. ROUNDS - 6 1/2" CTS.
TRANSVERSE BARS - 3/8" CORR. ROUNDS - 4 PER PANEL
1/2" CORR. ROUND BARS - 6 1/2" CTS.
21"
12"
SECTION SHOWING REINFORCEMENT IN FLOOR SLAB
STIRRUPS MADE OF 3/8" CORR. ROUNDS - SPACED AS SHOWN
G3
G4
1 BAR
2 BARS.
7-7/8" CORR. SQS. - 3 BENT UP AS SHOWN
14"x28" GIRDER.
1 BAR.
2 BARS
6-7/8" CORR. SQS. - 3 BENT UP AS SHOWN
SECTION SHOWING REINFORCEMENT IN GIRDERS
STIRRUPS MADE OF 3/8" CORR. ROUNDS AND SPACED AS SHOWN
B1
B5
28"
14"
5-7/8" CORR. SQS. - 2 BENT UP AS SHOWN.
12"x21" BEAM.
4-7/8" CORR. SQS. - 2 BENT UP AS SHOWN
SECTION SHOWING REINFORCEMENT IN BEAMS.


COLUMN AND FOOTING DETAILS
TYPICAL REINFORCED CONCRETE BUILDING
NOTE · ALL CORRUGATED SQUARES ARE NEW STYLE
ROOF
12"x12"
13"x24"
HEIGHT VARIES WITH SLOPE OF ROOF
5TH FLOOR
16"x16"
13"x24"
11'-0"
4TH FLOOR
20"x20"
13"x24"
11'-0"
3RD FLOOR
24"x24"
15"x24"
11'-0"
8-7/8" CORR ROUNDS
6-7/8" CORR ROUNDS
2ND FLOOR
27"x27"
19"x24"
15'-0"
8-1" CORR ROUNDS
6-1" CORR ROUNDS
1ST FLOOR
29"x29"
24"x24"
10'-0"
BASEMENT
DETAIL OF TYPICAL INTERIOR COLUMN
TYPICAL WALL COLUMN
20'-6"
EVERY 3RD BAR IN TOP BENT AS SHOWN
4'-0"
SECTION - COMBINED FOOTING.
11.42'
7.88'
EVERY 3RD BAR BENT
1" CORR SQS. AS SHOWN.
PLAN - COMBINED FOOTING.
SHOWING REINFORCEMENT IN TOP OF FOOTING
1" CORR SQS. · AS SHOWN
PLAN · COMBINED FOOTING
SHOWING REINFORCEMENT IN BOTTOM OF FOOTING
10'-4"
12" 26"
1" CORR SQS BOTH DIRECTIONS AS SHOWN
10'-4"
3'-6"
DETAIL OF TYPICAL INTERIOR FOOTING.

THE STRENGTH OF RECTANGULAR BEAMS.

In the following analysis for determining the resisting moment of a reinforced concrete beam, it will be assumed that the bond between the concrete and steel is sufficient to bring them into the necessary intimate stress relationship, and that proper and adequate provision for all web stresses has been made. While it is evident that simple formulæ are desirable, and that there is no objection to the use of strictly empirical formulæ, yet it is essential that the designer be familiar with the complete analysis, so that when using a simplified formula, he will be familiar with its limitations.

Tension has been considered as existing in the concrete up to that fiber in which the extension equals that to which plain concrete can be subjected without cracking. While the influence of the tension in the concrete, on the resisting moment, is large for very low unit stresses, yet its effect is negligible when the stresses due to ultimate, or even working, loads are considered. It was thought advisable to include it in the analysis for the sake of making the discussion complete mathematically.

By first assuming a definite law of variation between stress and strain, for the concrete, it is possible to readily evaluate the resisting moment of the section for any given percentage of reinforcement by further assuming the stress in the steel. The principle of invariability of plane sections, although it does not strictly obtain in the case of reinforced concrete beams, in conjunction with the requirement that the total tension must equal the total compression is sufficient to determine the position of the neutral axis. The resisting moment may then be obtained by taking moments about either the neutral axis or the centroid of compression or tension. The form of curve representing the stress-strain relation in the concrete was chosen after a thorough investigation of experimental data on deformation, and a study of the actual carrying capacity of beams, and, we believe, represents the actual conditions with sufficient exactness.

In the following discussion it is assumed that a section plane, before bending, is plane after bending. It is further assumed that the modulus of elasticity of concrete varies, its value decreasing as the stress increases, and that its instantaneous value may be represented by the tangent to a parabola.

To obtain an equation for a parabola that would represent the variations of the modulus, an inspection of a number of stress-strain diagrams was made, which led to the conclusion that if the compressive strength be taken as two-thirds of the stress corresponding to the observed deformation at failure, using the initial modulus of elasticity noted, that the parabola so obtained would represent closely the actual stress-strain diagram. The tensile stresses in the concrete, between the neutral axis and that plane at which the unit elongation has the limiting value λ_t, are considered in the discussion.

We have, then, for Rectangular Beams, the following discussion :

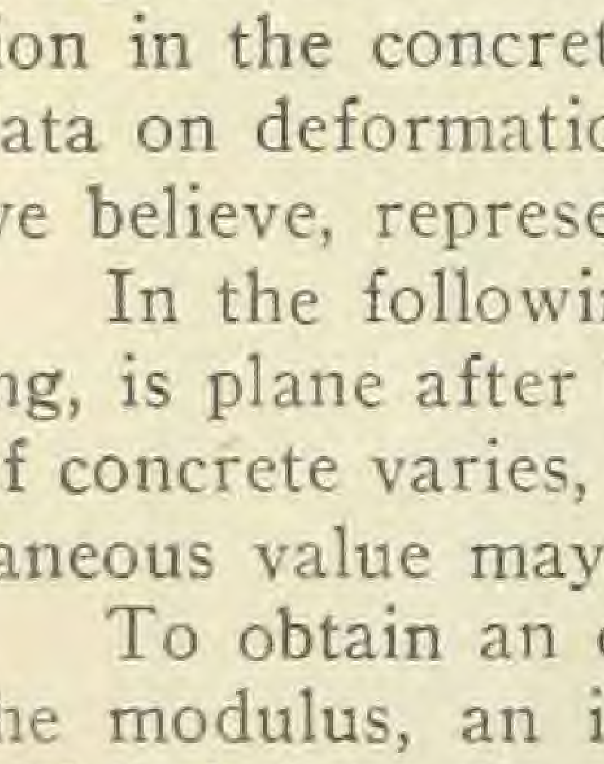


Figure 1.

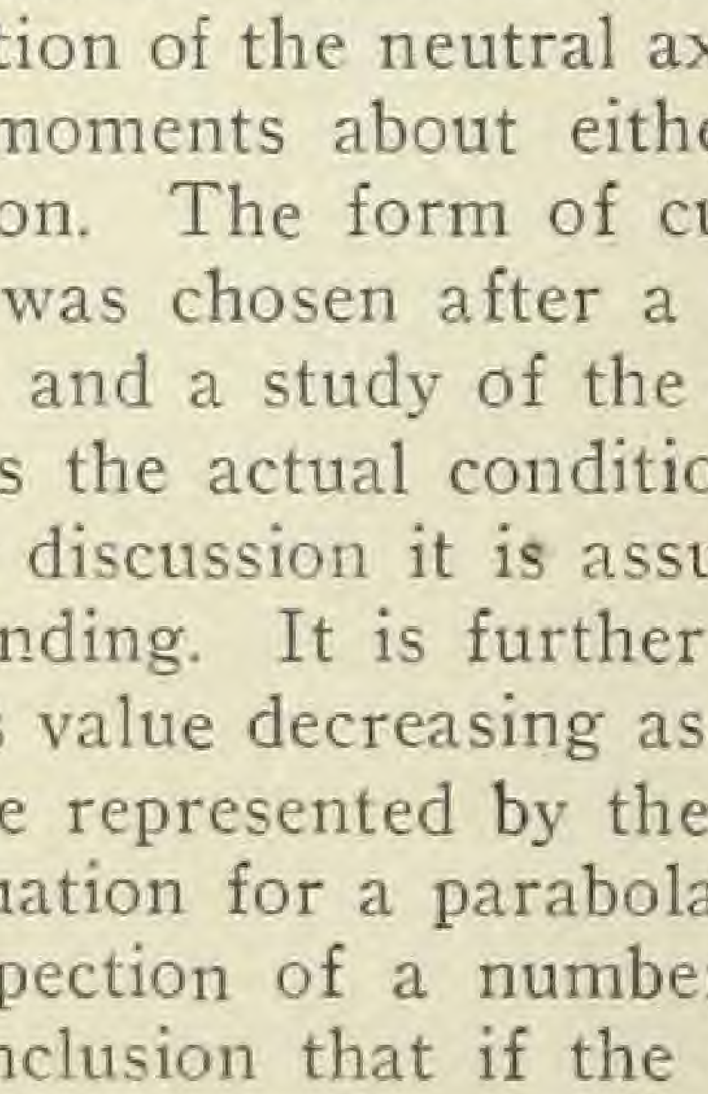


Figure 2.

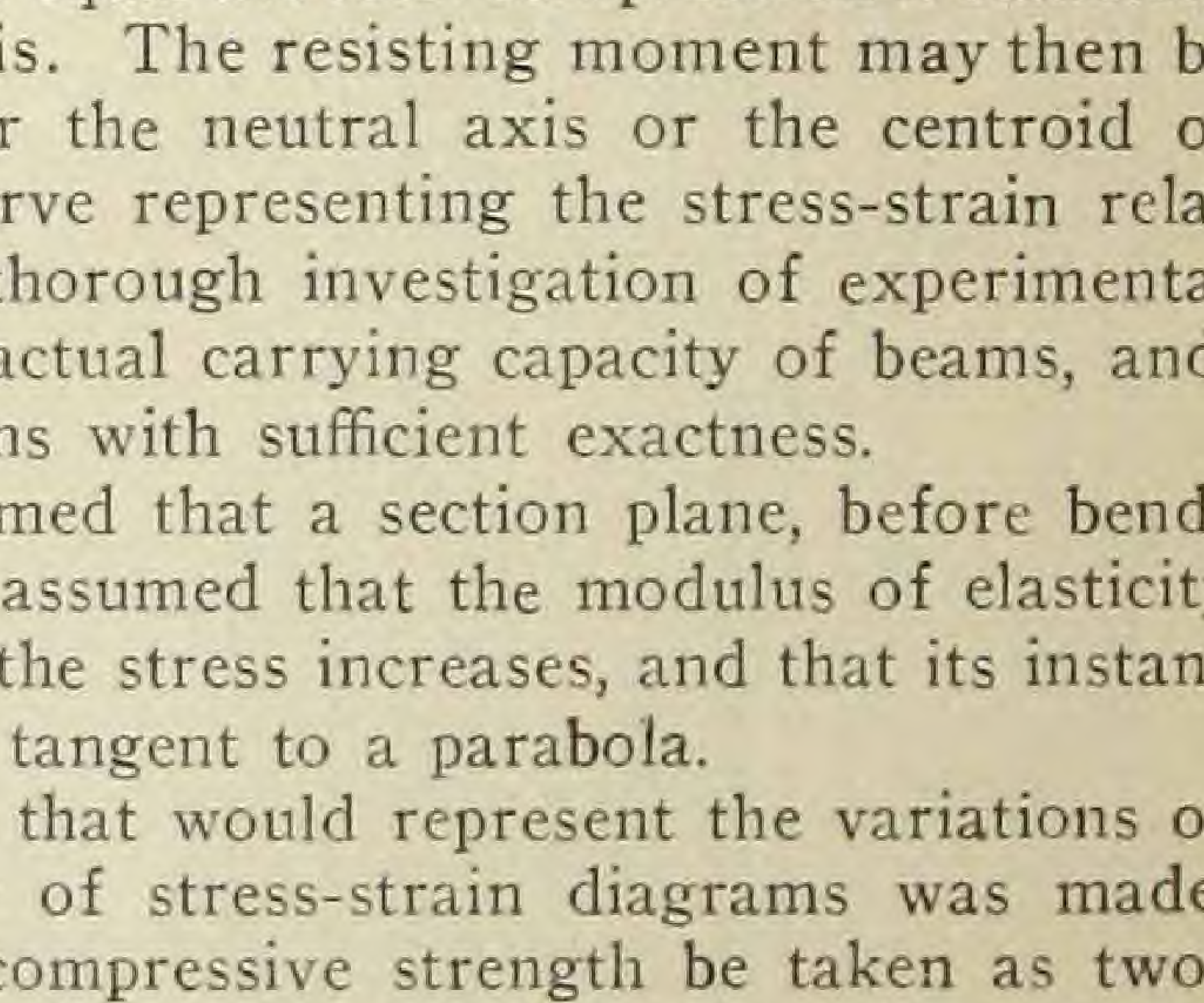


Figure 3.

Fig. 1 is a cross section of a reinforced concrete beam.

Fig. 2 represents the strain or deformation diagram at any instantaneous load.

Fig. 3 is the stress diagram corresponding to the above strain diagram.

Let E_s=Modulus of elasticity of steel in pounds per square inch.

E_c=Initial modulus of elasticity of concrete in compression in pounds per square inch.

F=Elastic limit of steel in pounds per square inch.

f_c=Compressive strength of concrete in pounds per square inch.

f=Compressive stress on extreme fiber in pounds per square inch. f may have any value less than fc.

c'=Abscissa to stress diagram at vertex of parabola.

s=Any assumed unit stress in steel, pounds per square inch.

f_t=Modulus of rupture of concrete in cross bending in pounds per square inch.

λ_c=Unit deformation of extreme compression fiber corresponding to a stress f.

λ_c''=Unit deformation of extreme compression fiber at ultimate stress fc.

λ_c'=Unit deformation corressponding to stress c'.

Note that λ'_c and c' deal with conditions after the ultimate strength of the concrete is passed, and have no value except in determining the curve, etc.

λ_t=Unit elongation of concrete corresponding to stress f_t.

λ_s=Unit elongation of steel corresponding to stress s.

b=Width of beam in inches.

z=Distance from top fiber to center of gravity of compression area in inches.

y_1=Distance from neutral axis to extreme fiber in compression in inches.

y_2=Distance from neutral axis to plane of reinforcement in inches.

e=Distance in inches from plane of metal to extreme fiber on tension side.

$d=y_1+y_2$=Effective depth of beam.

p=Ratio of reinforcement in terms of $bd=q\div bd$.

a=Ratio of reinforcement in terms of $bh=q\div bh$.

M=Bending moment of external forces in inch pounds=resisting moment of beam.

M_o=Ultimate moment of resistance of cross section in inch pounds.

P_s=Total stress in metal in width b.

P_c=Total compressive stress in concrete in width b.

P_t=Total tensile stress in concrete in width b.

q=Area of metal in width b, in square inches.

Referring to Fig. 3, the shaded area above the neutral axis represents the compressive stress diagram of the concrete, O Y being the axis of proportionate elongation, and O X the axis of stress per square inch.

Before getting the area of the compression diagram, it will be necessary to get the equation of the parabola referred to the axes O X and O Y. We have E_c which is represented by the tangent to the parabola at the origin O, and have also imposed the condition that the compressive stress at rupture is two-thirds $E_c\lambda_c''$. The equation for the parabola then becomes:

$$f = E_c\,\lambda_c - \frac{E_c\,\lambda_c^2}{2\lambda'_c}$$

$$f_c = \tfrac{2}{3}\,E_c\,\lambda_c''$$

From which $\lambda_c''=\dfrac{3\,f_c}{2\,E_c}$ ....(8)

And $\lambda_c'=\dfrac{9\,f_c}{4\,E_c}$ ....(9)

Substituting in the general equation and solving for λ_c, we get:

$$\lambda_c = \lambda_c' \left(1 - \sqrt{1 - \frac{8f}{9f_c}} \right) = \lambda_c' \, r \ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

We can now get a value for y_1 : —

From the strain diagram,

$$\frac{y_1}{d - y_1} = \frac{\lambda_c}{\lambda_s}$$

Or $$y_1 = \frac{\lambda_c d}{\lambda_s + \lambda_c}, \text{ but } \lambda_s = \frac{s}{E_s}$$

Therefore, $$y_1 = \frac{E_s \lambda_c}{s + E_s \lambda_c} d \ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

The expression for the area of the compressive stresses may be written in the form,

$$\frac{P_c}{b} = \left(1 - \frac{\lambda_c}{3 \lambda_c'} \right) \frac{E_c \lambda_c}{2} y_1 = D y_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

For the area of the tensile stresses we may, without appreciable error, consider the parabolic area as a triangle (since the allowed stress is very small, the tangent and parabola practically coincide), and can express the area by the equation,

$$\frac{P_t}{b} = \frac{f_t}{2} \frac{\lambda_t}{\lambda_c} y_1 = \frac{E_c \lambda_t^2}{2 \lambda_c} y_1 = G y_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

Since the sum of the compressive and tensile stresses must equal zero, we can write $P_s = P_c - P_t$

Therefore, $$p = \frac{y_1}{d} \times \frac{D\text{-}G}{s} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

We have, taking moments about the center of gravity of the compressive stresses, the following expression for the moment of resistance of the section:

$$M = P_s \, (d\text{-}z) + P_t \left(\frac{2 \lambda_t}{3 \lambda_c} y_1 + y_1 - z \right)$$

$$M = pbds(d\text{-}z) + by_1 \, G(y_1 + \tfrac{2}{3} y_1 \frac{\lambda_t}{\lambda_c} - z) \ldots\ldots\ldots\ldots\ldots (15)$$

$$z = \frac{4 - r}{12 - 4r} y_1, \text{ where } r = \frac{\lambda_c}{\lambda_c'} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

It is to be noted that the above discussion is perfectly general, and we may, by assuming any fiber stress, f, and any stress in the steel, s, find the percentage of reinforcement required, and the corresponding resisting moment of the section.

We are, however, mainly interested in the ultimate strength of the beam, reinforced with the critical percentage of metal (it being taken for granted that the designer will apply his factor of safety to actual moments, designing the section for the ultimate moment so obtained), which condition obtains when the percentage of steel is so chosen that the beam is equally strong in tension and compression; or, differently expressed, that the stress in the steel reaches the elastic limit at the same time that the compressive stress on the extreme fiber becomes the ultimate strength of the concrete.

Putting these values in the general equation, No. 15, we get the following:

$$M_o = pbd\,F(d\text{-}z) + by_1 \, G(y_1 + \tfrac{2}{3} y_1 \frac{\lambda_t}{\lambda_c} - z) \ldots\ldots\ldots (15a)$$

The size of beam needed to develop a required moment of resistance can be obtained from the above equations, when the constants dependent upon the particular materials used are known.

If it is desired to neglect the tension in the concrete, the last term in equation "15" and "15a" becomes zero, and we have:

For no tension in the concrete:

$$M = pbds\ (d\text{-}z) \ldots\ldots (17)$$

$$M_o = pbd\ F(d\text{-}z) \ldots\ldots (17a)$$

Note—If the extension of the steel corresponding to the assumed unit stress should happen to be less than the value for λ_t chosen, the formula would not apply. The value of λ_t will, in all cases, be taken as .00015; the unit stress in the steel corresponding to this extension being 4,350 pounds. If a lower stress were assumed in the steel λ_t would then have to be taken equal to the extension of the steel, λ_s

To illustrate the application of the formulæ, the following example has been worked out in detail:

Example—Determine the critical ratio of reinforcement and the ultimate moment of resistance of a beam, taking the compressive strength of the concrete as 2,000 pounds per square inch, and assuming the elastic limit of the steel to be 50,000 pounds per square inch.

For average rock concrete when fc=2,000 pounds, Ec may be taken as 2,600,000 and λ, =.00015. F is given as 50,000 pounds and Es may be taken as 29,000,000, and, in this particular case, where f=fc, λ_c becomes equal to λ_c''.

$$\text{From equation } (9)\lambda_c' = \tfrac{9}{4}\frac{f_c}{E_c} = \tfrac{9}{4} \times \frac{2000}{2{,}600{,}000} = .0017308$$

$$\text{Also } \lambda_c'' = \tfrac{2}{3}\lambda_c' = .0011539$$

$$r = \frac{\lambda_c}{\lambda_c'} = \tfrac{2}{3} \ldots\ldots (\text{Eq. } 16)$$

$$z = \frac{4 - r}{12 - 4r}\, y_1 = \tfrac{10}{28}\, y_1 \ldots\ldots (\text{Eq. } 16)$$

To obtain y_1, we substitute in equation 11,

$$y_1 = \frac{E_s\,\lambda_c}{s + E_s\,\lambda_c}\, d = \frac{33463}{83463} = .401\ d.$$

We will now obtain values for D and G, these quantities being respectively proportional to the amount of compression and tension in the concrete, only.

From equation 12:

$$D = \left(1 - \frac{\lambda_c}{3\,\lambda_c'}\right)\frac{E_c\,\lambda_c}{2} = (1 - \tfrac{1}{3}\,r)\,\frac{E_c\,\lambda_c}{2} = \tfrac{7}{9} \times \frac{2{,}600{,}000 \times .0011539}{2} = 1166.7$$

and from equation 13,

$$G = \frac{E_c\,\lambda_t^2}{2\,\lambda_c} = 25.4$$

The percentage of metal necessary may now be readily obtained by means of equation 14. This will be the critical ratio of reinforcement, since we are using fc and F as the stresses in the materials.

$$p = \frac{y_1}{d} \times \frac{D\text{-}G}{F} = .401 \times \frac{1143.3}{50000} = 0.00915$$

The ultimate moment of resistance may now be obtained by substituting the proper values in equation 15a.

$$M_o = pbd\ F(d\text{-}z) + by_1 G(y_1 + \tfrac{2}{3}\,y_1\,\frac{\lambda_t}{\lambda_c} - z) = 392\ bd^2 + 3\ bd^2 = 395\ bd^2$$

In a similar manner we obtain the following values for a good rock concrete beam, when fc may be taken as 2,700 pounds, and Ec= 2,800,000, F= 50,000.

$$\lambda_c'=.0021696$$
$$\lambda_c''=.0014464$$
$$p=.01418$$
$$M_o=596.6\ bd^2$$

Note—The average F for corrugated bars will be found to be about 55,000 pounds. With this value the critical ratio for average rock concrete becomes .00785, and for good rock concrete .01224.

It is desirable to have a ready means of determining the resisting moment of a section reinforced with a particular ratio of reinforcement, and the plates, Nos. 1 to 7, inclusive, give this information for various conditions.

Plates 1 and 3 represent graphically the results obtained by substituting in formula 15, the constants for the two grades into which we have arbitrarily divided all rock concretes. For average rock concrete, Plate 1, fc=2,000 and Ec= 2,600,000. For good rock concrete, Plate 3, fc=2,700 and Ec=2,800,000. In Plates 2 and 4 the effect of the tension in the concrete has been omitted. A comparison of Plate 1 with Plate 2, and of Plate 3 with Plate 4, shows that for high stresses in the steel the effect of the tension is negligible. For example, assuming a ratio of reinforcement of three-fourths of 1 per cent, and a stress in the steel of 50,000 per □″, we have:

From Plate 1; $M=328\ bd^2$ and $f=1840$ pounds.
From Plate 2; $M=326\ bd^2$ and $f=1825$ pounds.

Plates 5, 6 and 7 are based on the values for the constants shown, and in the computations the effect of the tension in the concrete has been omitted.

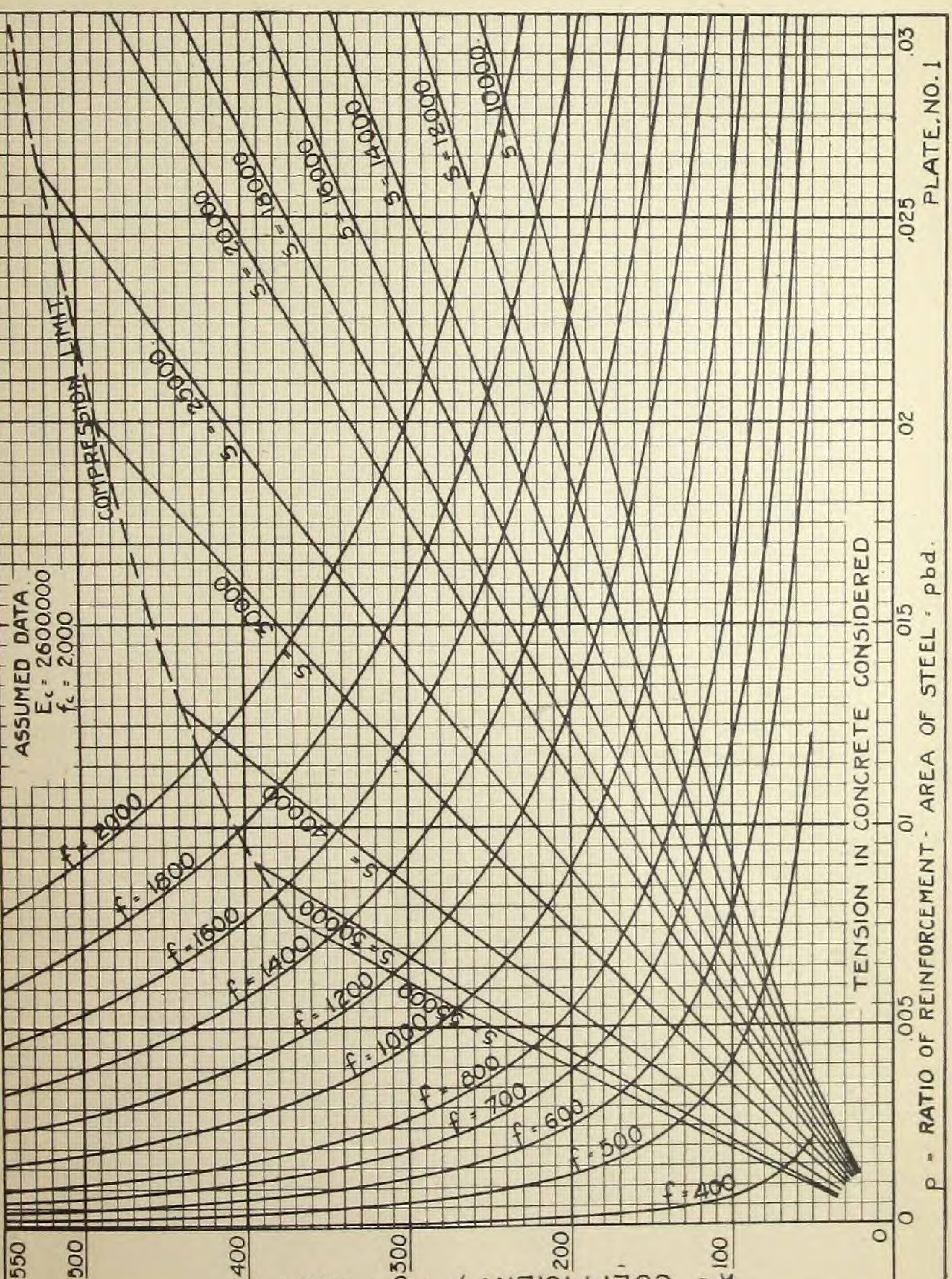
EXPLANATION OF PLATES.

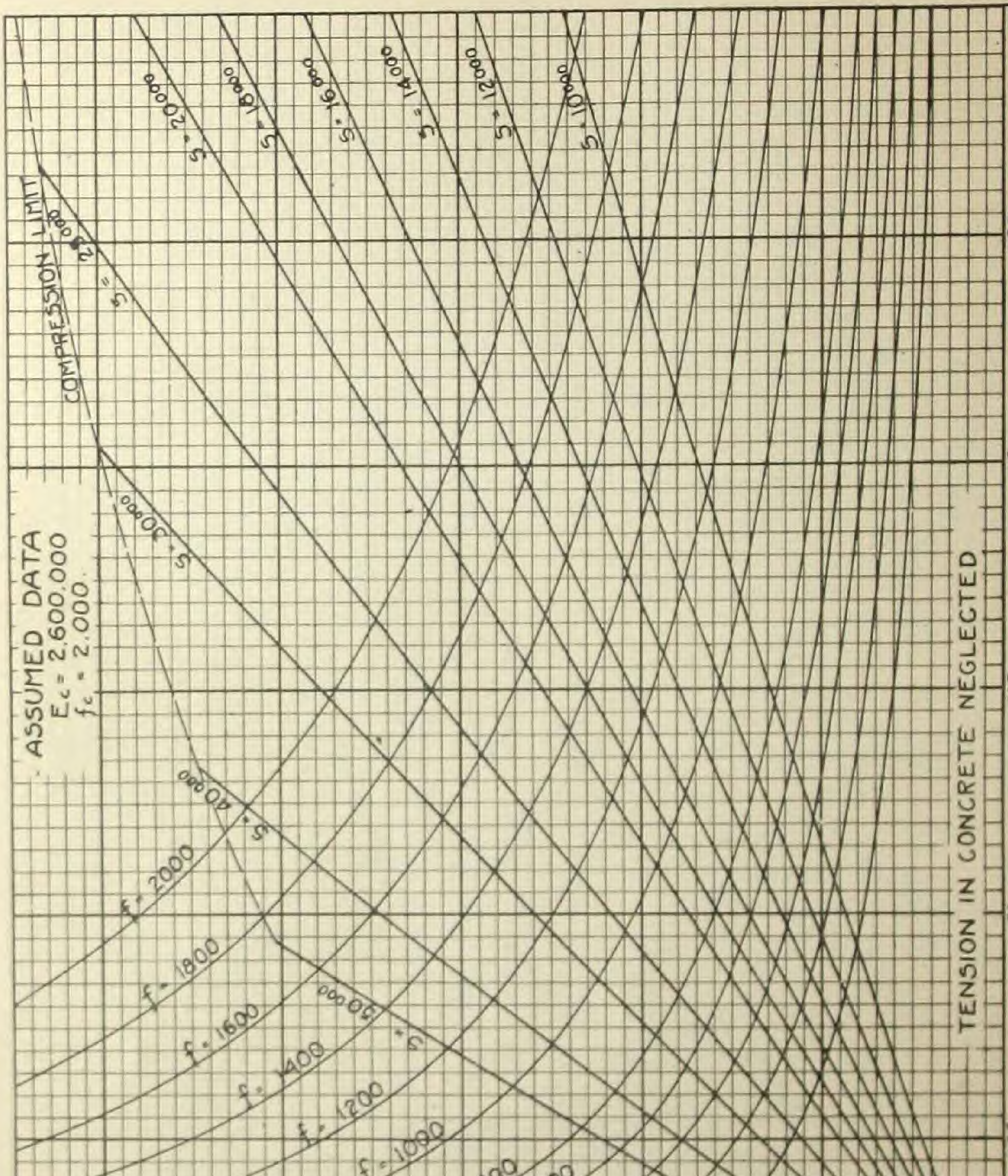
Each plate consists of two series of curves independent of each other, but with common abscissæ representing the ratio of reinforcement. The curved lines show the relation existing between p and s for the various values of f noted. For example, it is desired to develop 1,800 pounds extreme fiber-stress in average rock concrete when the stress in the steel is 50,000 pounds; what is the corresponding ratio, p, assuming tension to exist in the concrete?

Referring to Plate 1, follow the ordinate 50,000 until it intersects with the 1,800 curve; the ratio of reinforcement at this point is .007.

The straighter lines diverging from the origin give the relation between p and M for the various values of s given. Interpolate for values not given by curves. To get the moment of resistance of a section given p and s, we proceed as follows: Assume average rock concrete and include tension in the concrete. From Plate 1, follow up the line .007 = p to its intersection with the 50,000 curve. The ordinate of this point is 310 = K. Therefore, $M = 310\ bd^2$.

Conversely—If M and p are known, the stresses corresponding may be determined as follows: Let $M \div bd^2 = 300$, and take $p = .0075$; average rock concrete, assuming no tension. Referring to Plate 2: The point of intersection of the ordinate 300 with the abscissa .0075 is seen to be near the 50,000 curve. By interpolation its value may be estimated as 47,000 pounds. The stress in the concrete may now be found, since we have s and p. The intersection of the ordinate 47,000 and the abscissa .0075 is near the curve for $f = 1,800$ pounds, and by interpolation is seen to be about 1,750 pounds.


ASSUMED DATA
E_c = 2600,000
f_c = 2,000
COMPRESSION LIMIT
TENSION IN CONCRETE CONSIDERED
f = 2000
f = 1800
f = 1600
f = 1400
f = 1200
f = 1000
f = 800
f = 700
f = 600
f = 500
f = 400
s = 10000
s = 12000
s = 14000
s = 16000
s = 18000
p = RATIO OF REINFORCEMENT - AREA OF STEEL ÷ pbd.
STRESS IN STEEL - S
K = COEFFICIENT; M = Kbd²
PLATE. NO. 1


ASSUMED DATA
$E_c = 2,600,000$
$f_c = 2,000$
COMPRESSION LIMIT
TENSION IN CONCRETE NEGLECTED
s = 20,000
s = 18,000
s = 16,000
s = 14,000
s = 12,000
s = 10,000
s = 25,000
s = 30,000
s = 40,000
s = 50,000
f = 2000
f = 1800
f = 1600
f = 1400
f = 1200
f = 1000
f = 800
f = 700
f = 600
f = 500
f = 400
0
.005
.01
.015
.02
.025
.03
p = RATIO OF REINFORCEMENT ; AREA OF STEEL = bdp.
0
100
200
300
400
500
550
K = COEFFICIENT ; $M_o = Kbd^2$
0
10,000
20,000
30,000
40,000
50,000
55,000
STRESS IN STEEL = S
PLATE NO. 2

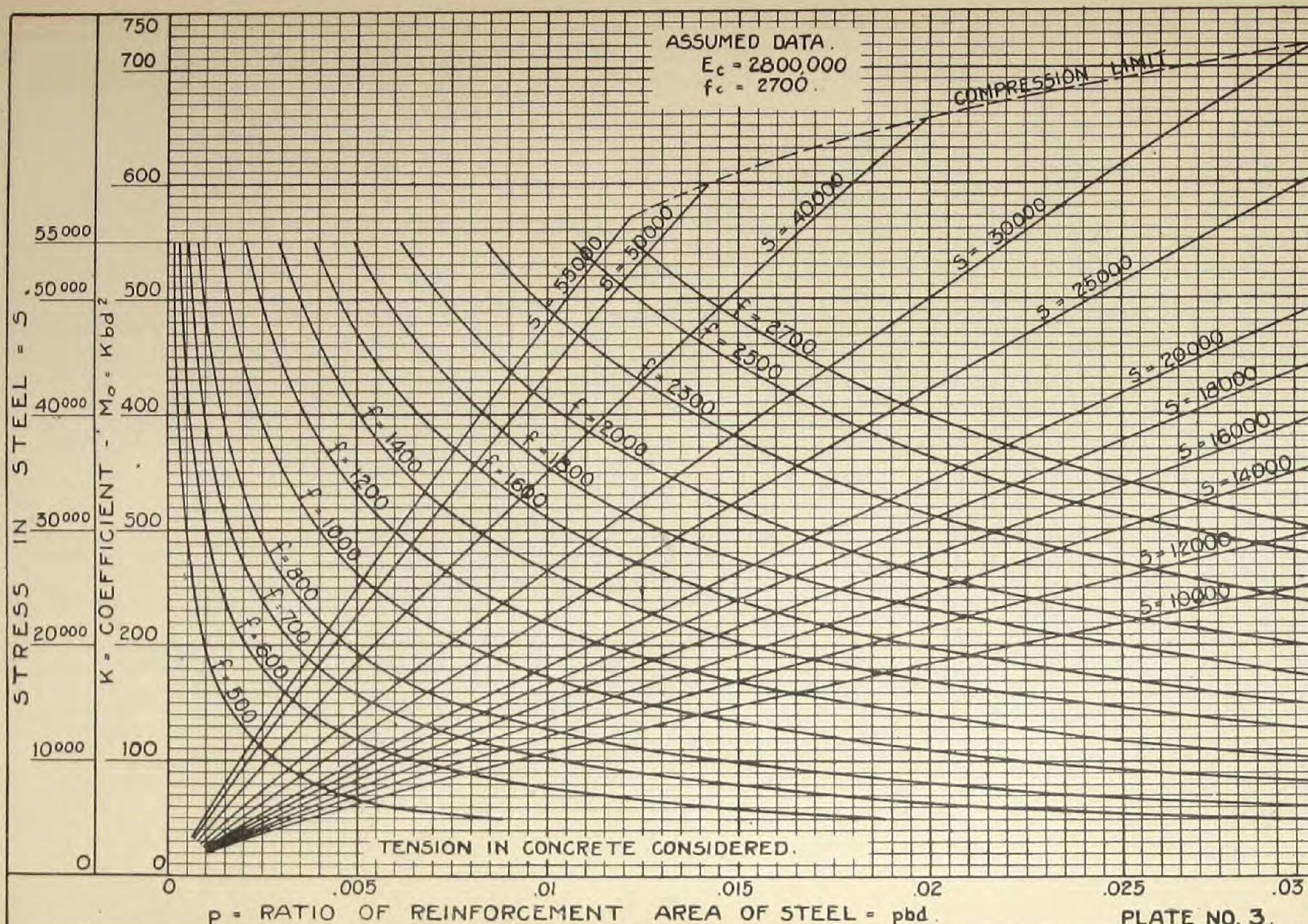
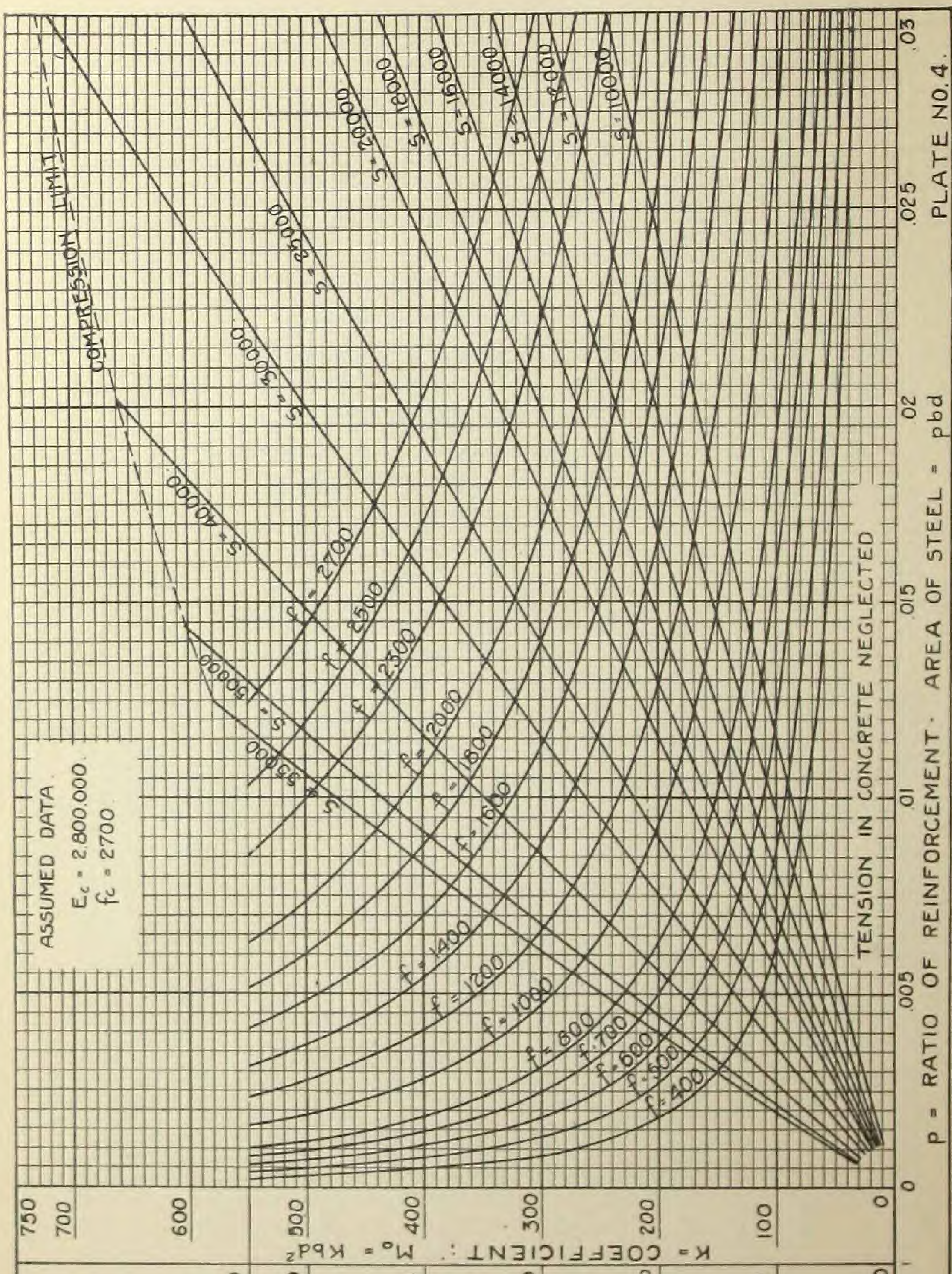
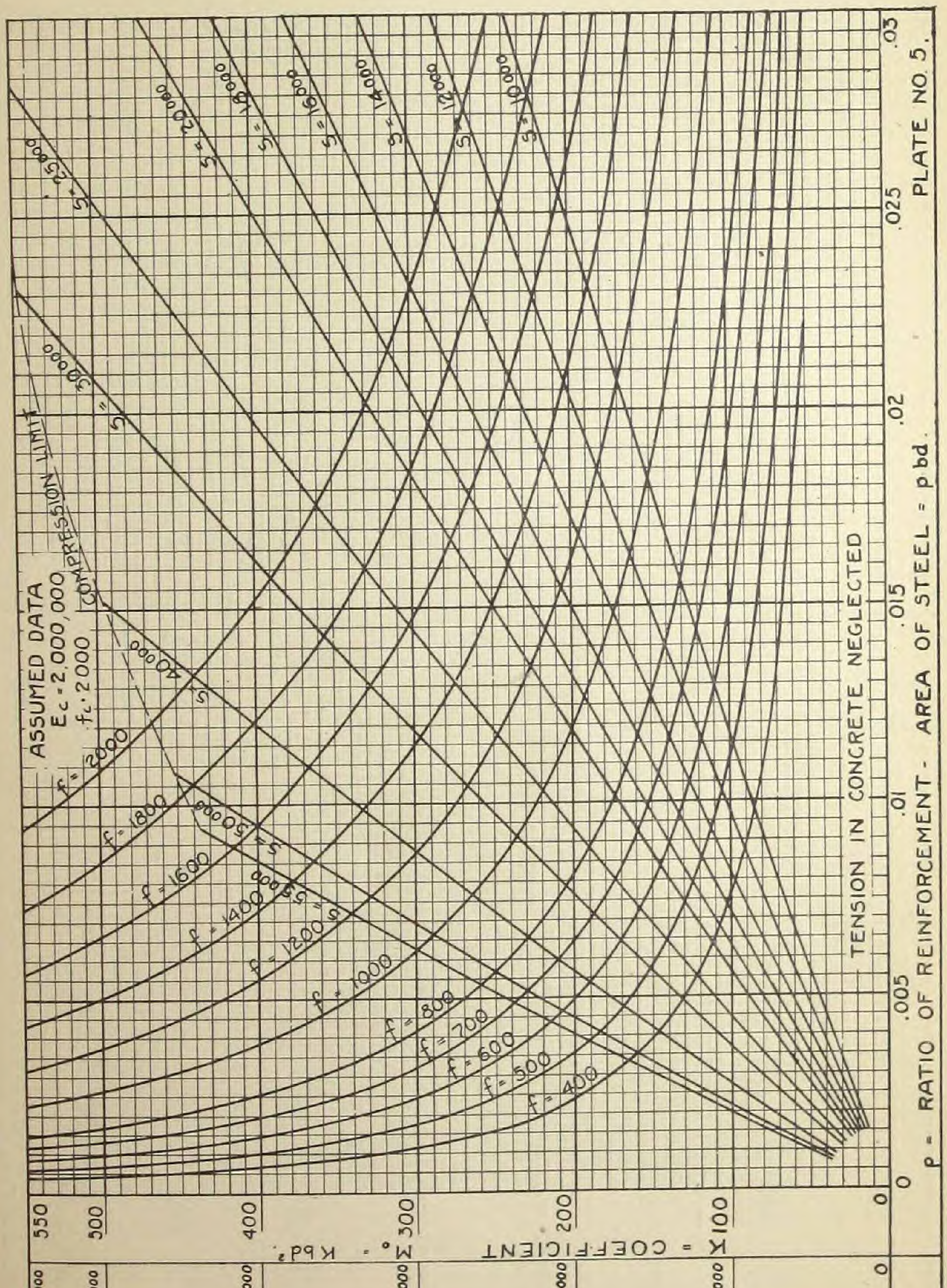

ASSUMED DATA.
E_c = 2800,000
f_c = 2700.
COMPRESSION LIMIT
TENSION IN CONCRETE CONSIDERED.
S = 55000
S = 50000
S = 40000
S = 30000
S = 25000
S = 20000
S = 18000
S = 16000
S = 14000
S = 12000
S = 10000
f = 2700
f = 2500
f = 2300
f = 2000
f = 1800
f = 1600
f = 1400
f = 1200
f = 1000
f = 800
f = 700
f = 600
f = 500
STRESS IN STEEL = S
K = COEFFICIENT - M_o = Kbd^2
p = RATIO OF REINFORCEMENT AREA OF STEEL = pbd
0
.005
.01
.015
.02
.025
.03
0
10000
20000
30000
40000
50000
55000
100
200
300
400
500
600
700
750

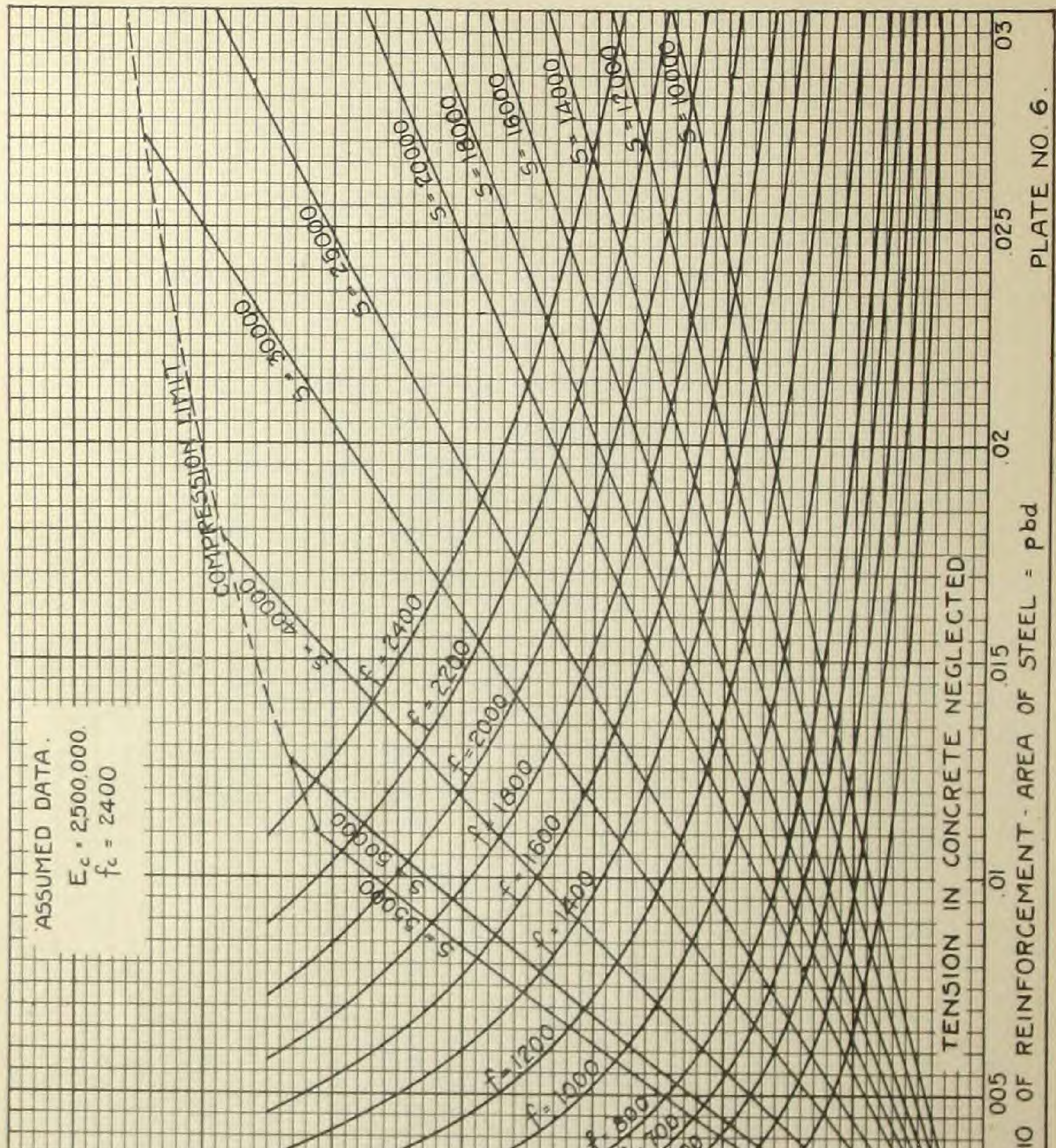
PLATE NO. 3.



ASSUMED DATA.
E_c = 2,800,000.
f_c = 2700
COMPRESSION LIMIT
TENSION IN CONCRETE NEGLECTED
S = 55000
S = 50000
S = 40000
S = 30000
S = 25000
S = 20000
S = 18000
S = 16000
S = 14000
S = 12000
S = 10000
f = 2700
f = 2500
f = 2300
f = 2000
f = 1800
f = 1600
f = 1400
f = 1200
f = 1000
f = 800
f = 700
f = 600
f = 500
f = 400
0
.005
.01
.015
.02
.025
.03
p = RATIO OF REINFORCEMENT · AREA OF STEEL = pbd
PLATE NO. 4
750
700
600
500
400
300
200
100
0
K = COEFFICIENT ∴ M_o = Kbd^2
55000
50000
40000
30000
20000
10000
0
STRESS IN STEEL = S

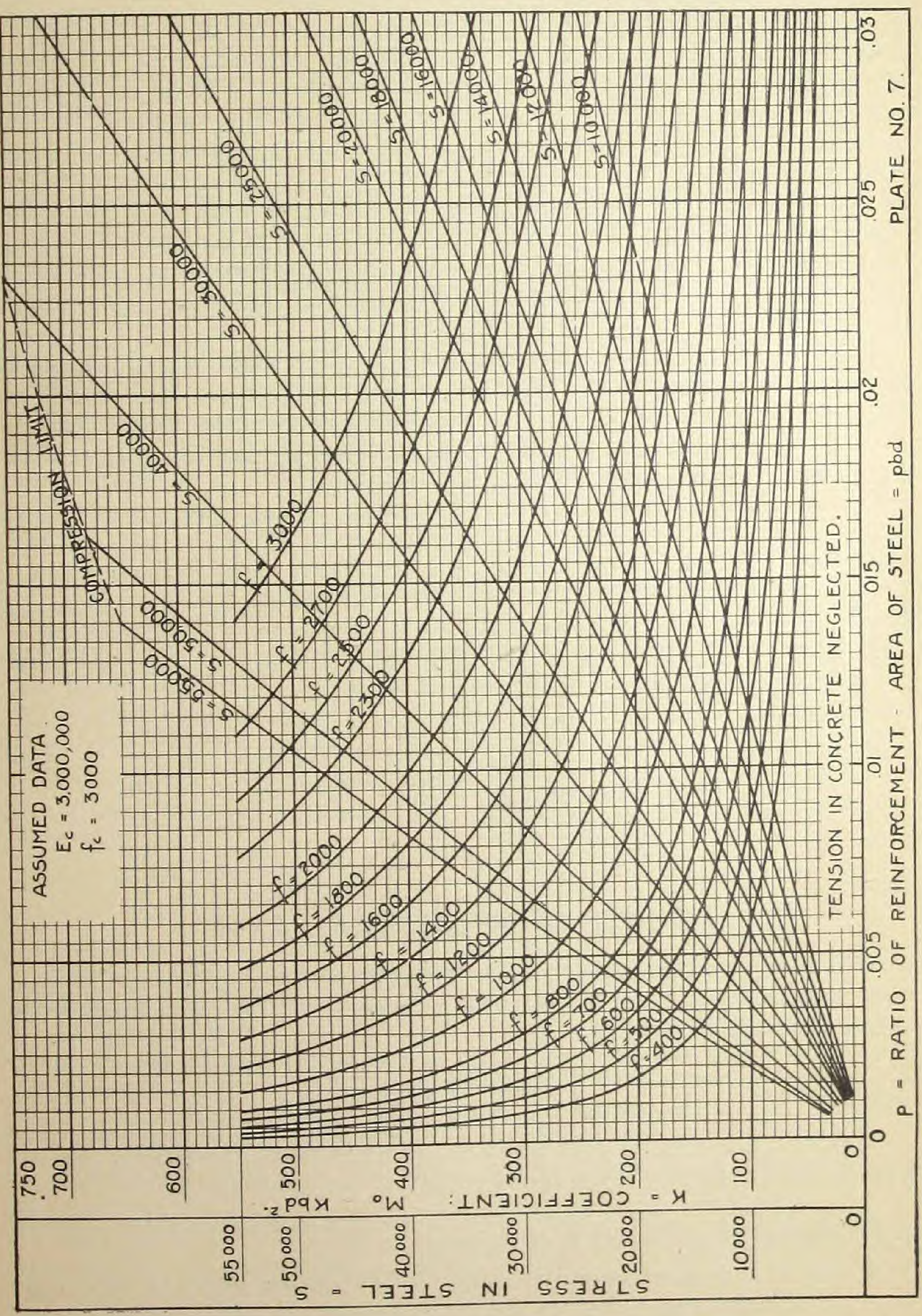


ASSUMED DATA
Ec = 2,000,000
fc = 2000
COMPRESSION LIMIT
TENSION IN CONCRETE NEGLECTED
p = RATIO OF REINFORCEMENT - AREA OF STEEL = p bd.
PLATE NO. 5.
STRESS IN STEEL = "S"
K = COEFFICIENT
Mo = K bd².
f = 2000
f = 1800
f = 1600
f = 1400
f = 1200
f = 1000
f = 800
f = 700
f = 600
f = 500
f = 400
S = 55000
S = 50000
S = 40000
S = 30000
S = 25000
S = 20000
S = 18000
S = 16000
S = 14000
S = 12000
S = 10000



ASSUMED DATA.
E_c = 2,500,000
f_c = 2400
COMPRESSION LIMIT
TENSION IN CONCRETE NEGLECTED
p = RATIO OF REINFORCEMENT - AREA OF STEEL = pbd
K = COEFFICIENT - Σ_0 = K bd²
STRESS IN STEEL - S
s = 55000
s = 50000
s = 40000
s = 30000
s = 25000
s = 20000
s = 18000
s = 16000
s = 14000
s = 12000
s = 10000
f = 2400
f = 2200
f = 2000
f = 1800
f = 1600
f = 1400
f = 1200
f = 1000
f = 800
f = 700
f = 600
f = 500
f = 400
0
.005
.01
.015
.02
.025
.03
750
700
600
500
400
300
200
100
55000
50000
40000
30000
20000
10000

PLATE NO. 6.


ASSUMED DATA
E_c = 3,000,000
f_c = 3000
COMPRESSION LIMIT
TENSION IN CONCRETE NEGLECTED.
S = 55000
S = 50000
S = 40000
S = 30000
S = 25000
S = 20000
S = 18000
S = 16000
S = 14000
S = 12000
S = 10 000
f = 3000
f = 2700
f = 2500
f = 2300
f = 2000
f = 1800
f = 1600
f = 1400
f = 1200
f = 1000
f = 800
f = 700
f = 600
f = 500
f = 400
p = RATIO OF REINFORCEMENT · AREA OF STEEL = pbd
0
.005
.01
.015
.02
.025
.03
STRESS IN STEEL = S
0
10000
20000
30000
40000
50000
55000
K = COEFFICIENT: M_o Kbd^2
0
100
200
300
400
500
600
700
750
PLATE NO. 7.

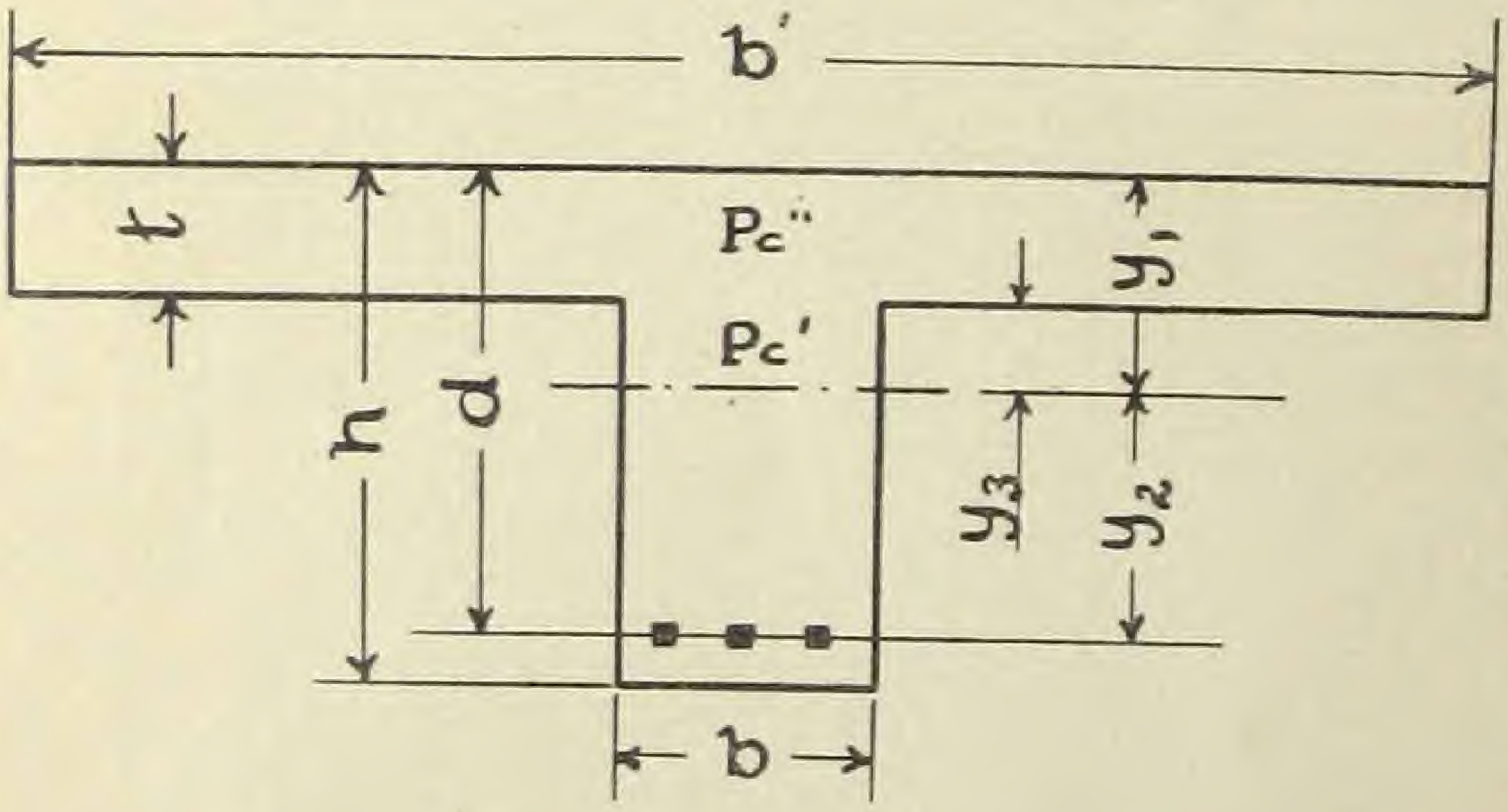
T-BEAMS.

In all T-Beam designs, when the ends of the beams are restrained, the strength of the section at the support should be investigated, and its ability to resist the negative moment there developed determined.

The resisting moment of a "T" section may be determined by simple approximate formulæ, and with sufficient accuracy for most purposes, when it is assured that the proportions of the beam are such that the integrity of the entire section will be preserved. Manifestly no more stress can be put into the flange than can be transmitted into it through the shearing strength of a horizontal section of the stem of a length equal to half the length of the beam, or into the wings than can be transmitted through the vertical planes of junction with the stem. An analysis based on these shearing values will then afford a means of determining the maximum amount of stress that can be developed in either tension or compression, and thus gives a means of determining the maximum amount of reinforcement that may be used.

The amount of compression being determined, the width of slab that should be considered tributary to the beam can be obtained, assuming some law of decrease in stress from the center of the beam to the edges of the wings.

It is for the purpose of bringing out these values and showing their relations more clearly that the following analysis is given:



In the analysis no account has been taken of any horizontal tensile stresses that may exist in the concrete.

LOCATION OF NEUTRAL AXIS.

The location of the neutral axis is dependent on the simultaneous deformations in the concrete and steel, and may be found, as in the case of rectangular beams, by means of equation 11.

$$y_1 = \frac{E_s \lambda_c}{s + E_s \lambda_c} d$$

Should the neutral axis be found to be in the flange, the analysis for rectangular beams applies strictly, b′ replacing b in the equations.

Investigation for failure by shearing or diagonal tension should be made in this case based on the width, b, of the stem.

Let S_v = Total shear in pounds along the two vertical planes of attachment between the wings and the beam from one end of the beam to the center of span.

S_h = Total shear in pounds along the horizontal plane of attachment of the flange and stem from one end of the beam to the center of span.

σ = Maximum shearing strength of concrete in pounds per square inch.

$K = \frac{y_3}{y_1}$

l = Length of span in feet.

P'_c = Compression existing at point of maximum bending moment, at ultimate load, in that portion of the section between the neutral axis and the underside of the flange.

P''_c = Compression existing at point of maximum bending moment at ultimate load in the flange.

b_0 = Limiting value of b'.

V = Total vertical shear at any section in pounds.

v = Intensity of vertical shear at any point in pounds per square inch.

d' = Distance from plane of reinforcement to centroid of compressive stresses, in inches.

All other functions as shown on the cut and in inches.

The sub-figure o denotes limiting or ultimate values.

We will now determine the ultimate resisting moment, M_0, of a T-section.

There are three methods of failure above the neutral axis:

1. By compression in the flange.
2. By deficiency in S_h owing to smallness of b.
3. By deficiency in S_v owing to smallness of t.

Condition 1 need not be considered, as the analysis which follows is based on a maximum stress of f_c in the extreme fiber of the concrete.

We will limit the discussion to a consideration of S_h, assuming the section to be so proportioned that S_v is greater than S_h.

The unit vertical shearing stress is a maximum at the end of a uniformly loaded beam, and its value decreases uniformly to zero at the center, and at any point the horizontal stress is equal in intensity to the vertical shearing stress. In a reinforced concrete beam, assuming the concrete to have no tensional value, the intensity of the shearing stress at any section is constant up to the neutral axis. Considering then, a uniformly loaded beam, the total shearing stress on the horizontal plane of junction of the stem from one end of the beam to the center = S_h.

$$S_h = 3\,\sigma\, b\, l\, P_o'' \quad \text{......} (18)$$

Similarly,

$$S_v = 6\,\sigma\, t\, l \quad \text{......} (19)$$

We can now get the necessary relation between b and t, such that we need consider failure through S_h only. For equal shearing strength, $t = \frac{1}{2}\,b$. But the stress transmitted into the wings through S_v is less than that transmitted through S_h by the amount of compression in that part of the flange directly over the stem. Also the floor slab reinforcement, running at right angles to the length of the beam, increases the shearing strength on the vertical planes. It may accordingly be safely assumed that if t is not less than $\frac{1}{3}$ of b, failure in S_v need not be apprehended. We will assume at once that t will not have a value less than $\frac{1}{3}\,b$.

The intensity of the vertical (and also horizontal) shear at any point below the neutral axis is given by the equation

$$v = \frac{V}{b\,d'} \quad \text{......} (20)$$

It is desirable to get an expression for S_h in terms of f_c. The theoretical relation between σ and f_c is: $\sigma = \frac{f_c}{2\tan\theta}$, where θ is the angle made by the

plane of rupture on a compression specimen, with a plane at right angles to the direction of the applied stress. (See "Johnson's Materials of Construction," page 19.) For concrete

$$\theta \text{ fs about } 60^\circ\text{, hence } \sigma = \frac{f_c}{3.5}$$

Although the above relation between compressive strength and shearing strength holds for homogeneous materials, we think it desirable to use a lower value for shearing strength in concrete than is given by this equation. The shearing strength is affected to a much greater extent by variations in uniformity of structure, than is the compressive strength. We will accordingly take twice the factor in shear that we do in direct compression, and derive the following value for Pc":

$$P_c'' = 3\,\sigma\, b\, l = \frac{1}{2} \times \frac{3\, f_c\, b\, l}{3.5} = .43\, b\, l\, f_c \quad \ldots\ldots\ldots\ldots\ldots (21)$$

If we replace the stress-strain diagram by a parabola, with its vertex at the top of the beam, and coinciding with the stress-strain diagram at this point and at the neutral axis, the area included by this parabola will closely approximate the actual stress-strain area. By using this parabolic area, we simplify the mathematics, and get results sufficiently accurate for T-beam analysis. We then have as a value for the compression in the stem:

$$P_c' = f_c\, y_1\, K^2 (1 - {}^1\!/_3\, K)\, b \ldots\ldots\ldots\ldots\ldots\ldots\ldots (22)$$

$$\text{and } P_c = P_s = P_c' + P_c'' = F\, q.$$

Taking moments around the plane of the metal we have:

$$M_o = P_c'(y_2 + {}^2\!/_3\, y_3) + P_c''\left(d - \frac{t}{2}\right) \ldots\ldots\ldots\ldots\ldots\ldots (23)$$

This M_o is the maximum moment of resistance of the section as determined from a consideration of the strength of the connection between flange and stem.

The maximum amount of reinforcement that may be used in a T-beam is, accordingly:

$$q_o = \frac{P_c' + P_c''}{F} = \frac{b\, f_c}{F}\left(y_1 K^2 (1 - {}^1\!/_3\, K) + .43\, l\right) \ldots\ldots (24)$$

To determine the width of flange that should be considered tributary to the beam so that the stress will be zero at the edges of the wings, it will be necessary to get P_c'' in terms of b_o. We have:

$$P_c'' = \frac{1}{3}(2 + K^3 - 3\, K^2) f_c b_o y_1$$

on the assumption that the stress is the same in the outer portion of the wings as in the middle; this would not be the case, the stress varying as the ordinates to a parabola from zero at a distance $\frac{b_o}{2}$ from the center line to a maximum over the stem. We must, therefore, take two-thirds of the above value for the stress in the wings, and doing so, obtain the following:

$$P_c'' = \frac{f_c\, b\, y_1}{3}(2 + K^3 - 3K^2) + \frac{2}{9}\left(f_c y_1 (2 + K^3 - 3K^2)(b_o - b)\right)$$

$$= \frac{f_c\, y_1}{9}(2 + K^3 - 3K^2)(b + b_o) \ldots\ldots\ldots\ldots\ldots\ldots (25)$$

And we have for b_o

$$b_o = \frac{P_c''}{\frac{2}{9} f_c\, y_1 (2 + K^3 - 3\, K^2)} - \frac{b}{2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (26)$$

$P_c'' = .43\, blf_c$, and substituting this value in equation 26 we have:

$$b_o = \frac{b}{2}\left(\frac{3.87l}{y_1(2 + K^3 - 3K^2)} - 1\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (27)$$

Problem—Required the size of T-beam necessary to carry a total ultimate load of 600 pounds per square foot on a span of 32′, ribs to be 9 feet apart.

$$M_o = \frac{12 \times 9 \times 600 \times 1024}{8} = 8{,}300{,}000\text{-inch pounds.}$$

For this spacing of beams the floor-slab should be 4″ thick, and we will assume d = 20″. Using good rock or gravel concrete, we have from (Eq 11), assuming F = 55,000 pounds:

$$\left.\begin{array}{l} y_1 = 8.66 \\ y_2 = 11.34 \\ K = .538 \\ K^2 = .289 \\ K^3 = .155 \end{array}\right\} \begin{array}{ll} P_c' = f_c\, y_1 K^2 \left(1 - \frac{K}{3}\right) b = 2700 \times 8.66 \times .289 \times .82b = & 5500b \\ P_c'' = .43\, blf_c = .43 \times 32 \times 2700 & = 37100b \\ P_c = P_s = Fq = P_c' + P_c'' & = 42600b \end{array}$$

$$M_o = P_c'(y_2 + \tfrac{2}{3} y_3) + P_c''\ (d - \tfrac{t}{2})$$

$$= 5500 \times 1755b + 37100 \times 18b = \begin{cases} 96500b + \\ 667800b \end{cases}$$

$$= 764300b$$

$$b = \frac{8300000}{764300} = 10.8''$$

$$q = \frac{42600 \times 10.8}{55000} = 8.4 \text{ square inches.}$$

$$b_o = \frac{P_c''}{\frac{2}{9} f_c\, y_1 (2 + K^3 - 3K^2)} - \frac{b}{2} = \frac{37100 \times 10.8}{\frac{2}{9} \times 2700 \times 8.66 \times 1.29} - \frac{10.8}{2} = 54.6''$$

FORMULAE OMITTING P_c'.

The neutral axis will, in the majority of T-beams, be found to be below the flange. It will also be seen by reference to the previous example that Pc′ is very small, and that its effect on the resulting resisting moment may be considered negligible.

We may then write, with sufficient accuracy for most purposes:

$$P_c'' = S_h = 3\,\sigma\, bl \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (28)$$

$$M_o = P_c''\ (d - \frac{t)}{2} = 3\,\sigma\, b\ l\ (d - \frac{t}{2}) \ldots\ldots\ldots\ldots\ldots\ldots (29)$$

The value of y_1 is obtained as before, from the equation

$$y_1 = \frac{Es\lambda_c}{S + Es\lambda_c}\, d$$

That value for σ, which applies to the particular concrete used, should be substituted in Eq (29). The value chosen, however, should not be more than one-seventh of the compressive strength of the concrete.

If we express σ in terms of f_c as before, we have

$$M_o = .43\ b\ l\ f_c\ (d - \frac{t}{2}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (30)$$

The maximum amount of metal that may be used $= q_o = \frac{P_c''}{F}$.

The value of b_o is obtained as before by means of Equation 26 or 27.

Applying these formulæ to the preceding example, we have:

$$b = \frac{8{,}300{,}000}{667{,}800} = 12.''5$$

$$q_o = \frac{37{,}100 \times 12.5}{55{,}000} = 8.4 \text{ sq. inches.}$$

REMARKS ON PRECEDING ANALYSIS.

In the analysis it is assumed that a sufficient width of slab (b_0) is available so that the compressive stress is zero at the edges. From the assumptions made b_0 as obtained from the equations is 50 per cent greater than the width required on the assumption of a uniform distribution of stress over the flange. It is evident that b_0 may be reduced quite considerably without appreciably affecting the strength of the beam, since the outer portions of the flange carry very little stress. When the width of flange is limited, it may, however, be necessary to make a corresponding reduction in P_c''.

EXPLANATION OF T-BEAM TABLE.

The following table will be found useful for the rapid design of T-sections. To illustrate the use of the table the preceding problem will be solved by its use.

The table is based on ultimate values; the compression in the concrete being limited to 2,700 pounds per square inch. The stress in the steel has been taken as 50,000 pounds per square inch:

Example:

$$M_0=8{,}300{,}000 \text{ inch lbs}; \; t=4'', \text{ and } l=32'\text{-}0''.$$

Assuming $d=20''$ as before, we have from table for 4″ slab,

$$M_0=b\,(90{,}300+20{,}900l)=8{,}300{,}000 \text{ inch lbs.}$$
$$=b\,(90{,}300+20{,}900\times 32)=759{,}100\,b.$$
$$\text{whence } b=\frac{8{,}300{,}000}{759{,}100}=10.9 \text{ in.}$$

Steel required

$$=b\,(0.1264+.0232l)=b\times 0.8688=9.45 \text{ sq. in.}$$
$$b_0=172\,bl-\frac{b}{2}=55\tfrac{1}{2}''.$$

Note—In the solution of the problem by the formula F was taken = 55,000 pounds. In this table a more general value: 50,000 pounds, has been adopted. If F had been assumed at the higher value, the amount of steel required would have been $\frac{50{,}000}{55{,}000}\times 9.45=8.5\ \square''$ This agrees with the amount before obtained.

TABLE FOR THE DESIGN OF TEE BEAMS.

GOOD ROCK CONCRETE. $S=50000$ $f_c=2700$ $P_c''=1161\ bl.$

t	d	AREA OF STEEL	ULTIMATE MOMENT	b_1
	10	b(.0122+.0232 l)	b(3750+ 9580 l)	.230bl—b/2
	11	b(.0222+.0232 l)	b(7810+10750 l)	.220bl—b/2
	12	b(.0337+.0232 l)	b(13200+11900 l)	.214bl—b/2
	13	b(.0464+.0232 l)	b(20160+13080 l)	.210bl—b/2
	14	b(.0600+.0232 l)	b(28600+14250 l)	.205bl—b/2
3½"	15	b(.0736+.0232 l)	b(38200+15400 l)	.202bl—b/2
	16	b(.0882+.0232 l)	b(49500+16550 l)	.200bl—b/2
	17	b(.1030+.0232 l)	b(62200+17700 l)	.198bl—b/2
	18	b(.1182+.0232 l)	b(76500+18850 l)	.196bl—b/2
	19	b(.1336+.0232 l)	b(92000+20050 l)	.196bl—b/2
	20	b(.1486+.0232 l)	b(108700+21200 l)	.194bl—b/2
	10	b(.0035+.0232 l)	b(1035+ 9290 l)	.217bl—b/2
	11	b(.0103+.0232 l)	b(3430+10450 l)	.205bl—b/2
	12	b(.0195+.0232 l)	b(7320+11610 l)	.196bl—b/2
	13	b(.0302+.0232 l)	b(12600+12780 l)	.190bl—b/2
	14	b(.0424+.0232 l)	b(19420+13920 l)	.186bl—b/2
	15	b(.0548+.0232 l)	b(27500+15100 l)	.183bl—b/2
4"	16	b(.0684+.0232 l)	b(37250+16250 l)	.179bl—b/2
	17	b(.0824+.0232 l)	b(48400+17400 l)	.177bl—b/2
	18	b(.0964+.0232 l)	b(60700+18600 l)	.175bl—b/2
	19	b(.1112+.0232 l)	b(74700+19750 l)	.173bl—b/2
	20	b(.1264+.0232 l)	b(90300+20900 l)	.172bl—b/2
	22	b(.1564+.0232 l)	b(125000+23200 l)	.170bl—b/2
	24	b(.1876+.0232 l)	b(166000+25500 l)	.168bl—b/2
	26	b(.2160+.0232 l)	b(209500+27900 l)	.168bl—b/2
	10	b(.00004+.0232 l)	b(11+ 9000 l)	.212bl—b/2
	11	b(.0027+.0232 l)	b(880+10160 l)	.195bl—b/2
	12	b(.0087+.0232 l)	b(3120+11320 l)	.185bl—b/2
	13	b(.0171+.0232 l)	b(6880+12500 l)	.177bl—b/2
	14	b(.0271+.0232 l)	b(12020+13650 l)	.172bl—b/2
	15	b(.0382+.0232 l)	b(18560+14800 l)	.168bl—b/2
	16	b(.0507+.0232 l)	b(26800+16000 l)	.164bl—b/2
4½"	17	b(.0633+.0232 l)	b(36200+17120 l)	.161bl—b/2
	18	b(.0770+.0232 l)	b(47200+18300 l)	.159bl—b/2
	19	b(.0909+.0232 l)	b(59500+19460 l)	.157bl—b/2
	20	b(.1050+.0232 l)	b(73200+20600 l)	.156bl—b/2
	22	b(.1342+.0232 l)	b(105000+22920 l)	.154bl—b/2
	24	b(.1650+.0232 l)	b(143000+25250 l)	.152bl—b/2
	26	b(.1960+.0222 l)	b(187000+27600 l)	.151bl—b/2
	28	b(.2268+.0232 l)	b(235400+29900 l)	.150bl—b/2
	30	b(.2580+.0232 l)	b(290000+32200 l)	.149bl—b/2
	12	b(.0021+.0232 l)	b(725+11050 l)	.178bl—b/2
	13	b(.0073+.0232 l)	b(2830+12200 l)	.169bl—b/2
	14	b(.0156+.0232 l)	b(6430+13350 l)	.162bl—b/2
	15	b(.0246+.0232 l)	b(11550+14500 l)	.157bl—b/2
	16	b(.0350+.0232 l)	b(17900+15700 l)	.153bl—b/2
	17	b(.0465+.0232 l)	b(25750+16850 l)	.150bl—b/2
	18	b(.0590+.0232 l)	b(35200+18000 l)	.147bl—b/2
5"	19	b(.0718+.0232 l)	b(45800+19200 l)	.145bl—b/2
	20	b(.0854+.0232 l)	b(58200+20300 l)	.143bl—b/2
	22	b(.1130+.0232 l)	b(86700+22650 l)	.141bl—b/2
	24	b(.1426+.0232 l)	b(121500+25000 l)	.139bl—b/2
	26	b(.1726+.0232 l)	b(161700+27300 l)	.137bl—b/2
	28	b(.2036+.0232 l)	b(208000+29600 l)	.136bl—b/2
	30	b(.2344+.0232 l)	b(259000+31900 l)	.135bl—b/2
	32	b(.2660+.0232 l)	b(316200+34250 l)	.134bl—b/2
	34	b(.2980+.0232 l)	b(380000+36600 l)	.134bl—b/2
	14	b(.0062+.0232 l)	b(2560+13050 l)	.156bl—b/2
	15	b(.0132+.0232 l)	b(6000+14200 l)	.150bl—b/2
	16	b(.0220+.0232 l)	b(10900+15400 l)	.145bl—b/2
	17	b(.0318+.0232 l)	b(17100+16550 l)	.141bl—b/2
	18	b(.0429+.0232 l)	b(24800+17700 l)	.138bl—b/2
	19	b(.0550+.0232 l)	b(34200+18900 l)	.136bl—b/2
	20	b(.0673+.0232 l)	b(44600+20000 l)	.133bl—b/2
5½"	22	b(.0936+.0224 l)	b(70000+22350 l)	.130bl—b/2
	24	b(.1216+.0232 l)	b(101500+24700 l)	.128bl—b/2
	26	b(.1510+.0232 l)	b(138800+27000 l)	.127bl—b/2
	28	b(.1816+.0232 l)	b(182200+29300 l)	.125bl—b/2
	30	b(.2120+.0232 l)	b(230800+31600 l)	.124bl—b/2
	32	b(.2420+.0232 l)	b(284000+34000 l)	.123bl—b/2
	34	b(.2736+.0232 l)	b(344000+36300 l)	.122bl—b/2
	36	b(.3050+.0232 l)	b(410000+38600 l)	.122bl—b/2

TABLE FOR THE DESIGN OF TEE BEAMS—Continued.

GOOD ROCK CONCRETE. S=50000 f_c=2700 P_c''=1161bl.

t	d	AREA OF STEEL	ULTIMATE MOMENT	b_1
6″	16	$b(.0118+.0232\,l)$	$b(5630+15100\,l)$	$.139bl-b/2$
	17	$b(.0198+.0232\,l)$	$b(10300+16250\,l)$	$.134bl-b/2$
	18	$b(.0292+.0232\,l)$	$b(16400+17420\,l)$	$.131bl-b/2$
	19	$b(.0398+.0232\,l)$	$b(24100+18600\,l)$	$.128bl-b/2$
	20	$b(.0510+.0232\,l)$	$b(33050+19750\,l)$	$.125bl-b/2$
	22	$b(.0756+.0232\,l)$	$b(55400+22050\,l)$	$.122bl-b/2$
	24	$b(.1028+.0232\,l)$	$b(84000+24400\,l)$	$.120bl-b/2$
	26	$b(.1304+.0232\,l)$	$b(117500+26700\,l)$	$.118bl-b/2$
	28	$b(.1592+.0232\,l)$	$b(157000+29050\,l)$	$.116bl-b/2$
	30	$b(.1892+.0232\,l)$	$b(202500+31400\,l)$	$.115bl-b/2$
	32	$b(.2190+.0232\,l)$	$b(253000+33700\,l)$	$.114bl-b/2$
	34	$b(.2506+.0232\,l)$	$b(311000+36000\,l)$	$.113bl-b/2$
	36	$b(.2820+.0232\,l)$	$b(374000+38300\,l)$	$.113bl-b/2$
	38	$b(.3130+.0232\,l)$	$b(441500+40700\,l)$	$.112bl-b/2$
7″	16	$b(.0007+.0232\,l)$	$(b\ 294+14520\,l)$	$.133bl-b/2$
	17	$b(.0038+.0232\,l)$	$(b\ 1860+15700\,l)$	$.127bl-b/2$
	18	$b(.0091+.0232\,l)$	$(b\ 4820+16830\,l)$	$.122bl-b/2$
	19	$b(.0161+.0232\,l)$	$(b\ 9200+18000\,l)$	$.118bl-b/2$
	20	$b(.0244+.0232\,l)$	$(b\ 15000+19150\,l)$	$.115bl-b/2$
	22	$b(.0444+.0232\,l)$	$(b\ 31100+21500\,l)$	$.110bl-b/2$
	24	$b(.0674+.0232\,l)$	$(b\ 52800+23800\,l)$	$.106bl-b/2$
	26	$b(.0930+.0232\,l)$	$(b\ 80800+26100\,l)$	$.104bl-b/2$
	28	$b(.1196+.0232\,l)$	$(b114000+28450\,l)$	$.102bl-b/2$
	30	$b(.1470+.0232\,l)$	$(b152500+30750\,l)$	$.101bl-b/2$
	32	$b(.1760+.0232\,l)$	$(b198000+33100\,l)$	$.100bl-b/2$
	34	$b(.2064+.0232\,l)$	$(b249500+35400\,l)$	$.099bl-b/2$
	36	$b(.2360+.0232\,l)$	$(b305000+37700\,l)$	$.098bl-b/2$
	38	$b(.2660+.0232\,l)$	$(b368000+40100\,l)$	$.097bl-b/2$
	40	$b(.2970+.0232\,l)$	$(b434000+42400\,l)$	$.097bl-b/2$
8″	20	$b(.0071+.0232\,l)$	$b(4140+18600\,l)$	$.109bl-b/2$
	22	$b(.0206+.0232\,l)$	$b(13720+20900\,l)$	$.102bl-b/2$
	24	$b(.0390+.0232\,l)$	$b(29300+23200\,l)$	$.098bl-b/2$
	26	$b(.0604+.0232\,l)$	$b(50600+25500\,l)$	$.095bl-b/2$
	28	$b(.0844+.0232\,l)$	$b(77600+27900\,l)$	$.093bl-b/2$
	30	$b(.1096+.0232\,l)$	$b(110200+30200\,l)$	$.091bl-b/2$
	32	$b(.1368+.0232\,l)$	$b(149200+32500\,l)$	$.090bl-b/2$
	34	$b(.1646+.0232\,l)$	$b(193300+34800\,l)$	$.088bl-b/2$
	36	$b(.1928+.0232\,l)$	$b(243000+37200\,l)$	$.087bl-b/2$
	38	$b(.2224+.0232\,l)$	$b(299000+39500\,l)$	$.086bl-b/2$
	40	$b(.2526+.0232\,l)$	$b(361000+41800\,l)$	$.086bl-b/2$
	42	$b(.2820+.0232\,l)$	$b(426500+44200\,l)$	$.085bl-b/2$
	44	$b(.3130+.0232\,l)$	$b(500000+46500\,l)$	$.085bl-b/2$
	46	$b(.3430+.0232\,l)$	$b(578000+48750\,l)$	$.084bl-b/2$
	48	$b(.3760+.0232\,l)$	$b(664000+51100\,l)$	$.084bl-b/2$

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